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ABSTRACT

Particle size distribution (PSD) is recognised among geotechnical engineers as an informative soil descriptor, and often used to predict geomechanical behaviours. However, the effectiveness of PSD to characterise frost action is debatable. Existing criteria for assessing frost susceptibility have relied on traditional PSD descriptors, such as C_u, which depend on individual parameters (i.e., d_{10}, d_{60}) which explicitly neglect the effect of fines and gravel content. In turn, it has been reported that fines content is critical in the formation of ice lenses. Grading entropy is a method which accounts for all the information in the PSD. In this work, normalised entropy coordinates are used to review PSD-based frost susceptibility criteria and assess whether alternative PSD descriptors can more successfully characterise frost action susceptibility. The effect of PSD (via grading entropy coordinates) on the development of frost heave is investigated using existing experimental datasets. The findings in this work highlight significant variability in the PSD criteria, suggesting that PSD alone is not a reliable indicator. However, examining experimental datasets indicated a clear effect of PSD using grading entropy coordinates for understanding the development of frost heave.

INTRODUCTION

Frost action in soils is one of the main factors contributing to road and foundation damage in cold regions. The term frost action is one which describes changes in soil structure as a result of the freeze-thaw cycle (Shoop, 2020). Two behaviours are involved in this process: frost heave and thaw weakening. Both behaviours incur damage to infrastructure. Due to the frequency of frost related geotechnical behaviours in cold regions, significant capital is invested annually into maintenance of frost damaged structures such as roads, pavements, and foundations (Shoop, 2020). Multiple efforts have attempted to establish criteria to predict whether a soil is susceptible to frost action. One of the earliest and most cost-effective methods of susceptibility assessment is the analysis soil particle size distribution (PSD) (Chamberlin, 1981).

This method offers a simple and quick way of assessing frost susceptibility. However, some research (e.g., Dagli, 2017; Sheng, 2021, Hao et al. 2023) suggest that PSD may not be an effective means of determining frost susceptibility. Moreover, many PSD based criteria have relied on traditional descriptors such as the uniformity coefficient (C_u), as used by Riss (1948), Jesseberger (1976) and Konrad and Morgenstern (1981), or the average grain size (d_{50}) as used by Beskow (1935). More elaborate methods including pore size distribution, have been also used by Csathy and Townsend (1963) and Reed et al. (1979). However, it is not clear how the effect of fines (Fc) or gravel content is explicitly considered in these methods. Fines content has been shown to be play a crucial role in the formation of ice lenses (e.g., Ćwiąkała et al. 2016; Niggemann and Fuentes, 2023). The coefficient of uniformity (C_u) in particular 'scalps' the PSD curve, excluding 50% of the PSD. This has implications not only on susceptibility criteria, but other parameters that may be used to assess PSD related behaviours within soils. Moreover methods to obtain the pore size distribution are often complex and may lead to creating "idealised freezing conditions" which may not be representative of in-situ soils (Sheng, 2021).

Due to the ease of obtaining PSD information, there is an incentive to explore the reliability of criteria that rely on it. Therefore, a review of PSD based frost susceptibility criteria is needed. To do so, the entirety of the PSD curve must be assessed. In this study, grading entropy coordinates are used to account for all the information within the PSD. The coordinates are used to compare previously proposed frost susceptibility criteria available in the Review of Frost Susceptibility Index Tests undertaken by Chamberlin (1981). Furthermore, two more recent experimental datasets from Bilodeau et al. (2008) and Hao et al. (2023) are used to investigate the effect of PSD on frost heave development.

GRADING ENTROPY

Grading entropy, proposed by Lőrincz (1986), condenses the entirety of a PSD to a single point on a Cartesian plane. This is achieved by accumulating the information entropy (Shannon, 1948) within each soil fraction to ascertain the total grading curve entropy. The total entropy (Eq. 1) can then be split into two components which form a coordinate pair:

$$S = \Delta S + S_0 \tag{1}$$

where S is the total grading entropy, ΔS is the entropy increment, and S_0 is the base entropy. The entropy increment (ΔS), normally plotted on the y-axis, is defined as:

$$\Delta S = -\frac{1}{\ln(2)}\sum_{i=1}^{N} x_i ln\, x_i \tag{2}$$

where N is the number of fractions between the finest and coarsest particles and x_i is the relative frequency, which relates to the mass retained within each sieve/fraction. The base entropy (S_0), normally represented on the x-axis, is given by:

$$S_0 = \sum_{i=1}^{N} x_i S_{oi} \tag{3}$$

where S_{oi} is known as the intrinsic entropy (Nadji et al. 2012). This is an integer which increases relative to standard sieve sizes, according to the Unified Soil Classification System (USCS). Soil fractions are numbered by increasing integers, as seen in Table 1, where 'd' is the sieve mesh diameter and S_{0i} the intrinsic entropy:

In Table 1, the limiting diameter values for the i^{th} fraction are given in terms of d_{min} as follows:

$$2^{i+1}d_{min} \geq d > 2^i d_{min}$$

Table 1. Fractions, their numbering and equivalent eigen-entropies, S_{oi} (modified after Barreto et.al 2019).

Fraction	0	…	22	23	24
d (mm)	2^{-22} - 2^{-21}	…	1 - 2	2 - 4	4 - 8
S_{0i}	0	…	22	23	24

For the purpose of grading entropy, the minimum grain diameter (d_{min}) is generally selected as the theoretical minimum grain size: the height of the SiO_4 tetrahedron, *i.e.*, 2^{-22} *mm*, Imre (1995). These coordinates have simple physical meanings. The base entropy (S_0) is an expected value of the probability distribution, in other words, it is a logarithmic mean of the average grain diameter and relates to the skewness of the PSD. The entropy increment (ΔS) is a measure of how much a PSD is influenced by all its fractions and relates to the kurtosis of the PSD. The ΔS-coordinate obtains the average information in each fraction (i.e. mass retained), ensuring that the influence of the fractions is sensitive to subtle changes in the PSD curve (Leak et al. 2022). Note that ΔS may be related to C_u, as they both quantify the PSD span. However ΔS considers the entire grading curve, making ΔS a more comprehensive descriptor. Note that ΔS and S_0 have normalised counterparts, where $0<A<1$ and $0<B<B_{max} \cong 1.41$. Here, A is the normalised base entropy and B is the normalised entropy increment, given by:

$$A = \frac{S_0 - S_{0min}}{S_{0max} - S_{0min}} \tag{4}$$

$$B = \frac{\Delta S}{\ln(N)} \tag{5}$$

Whilst grading entropy coordinates may be related to commonly used descriptors, it should be noted that in contrast to these, ΔS, S_0 and A, B involve information about the entire PSD, not just specific particles diameters (e.g., d_{10}, d_{30}, d_{50}, d_{60}) and enables for the direct consideration of both fines and gravel content with no need for additional descriptors. In this study, only the normalised coordinates are used, this is to ensure that the frost heave susceptibility is investigated with reference to the grading entropy stability criteria (Lorincz, 1986). Given that a more extensive definition of the stability criteria has been given elsewhere (e.g. Nadji et al. 2012; Imre et al 2012; Barreto et al, 2019), only a definition of the unstable zone is provided with reference to Figure 1.

Figure 1 illustrates the normalised entropy diagram. The transition from a stable coarse grained soil skeleton to a fines matrix will take place at A=2/3. As a PSD moves to the left from this stability line at A=2/3 (0.667), coarser grained soils become unstable and 'float' in a matrix of fines rendering them prone to internal instability. Behaviours in the unstable zone have been verified on the basis of experimental data (e.g. Nadji et al. 2012; Barreto et al. 2019; Lorincz, 1986).

REVIEW OF EXISTING PSD CRITERIA

In addition to the stability limits, Figure 1 presents PSD based frost susceptibility limits from criteria proposed by: Beskow, (1935), Corothors (1948), Croney (1949), Armstrong and Csathy

(1963), Nielsen Rayschenberger (1952), Pietrzyk (1980), Jessberger (1969), Hartel (1967), and Riis (1948) as individual points. Grading entropy coordinates for each PSD have been chosen and included here on the basis of the Frost Susceptibility Index Tests (Chamberlin, 1981). It is important to note that in this present study soils were classified as susceptible or non-susceptible. In other words, if a criteria characterises a soil as susceptible then it has been labelled so in Figure 1. Therefore the data in Figure 1 may be used to assess common relationships in susceptibility between criteria, PSD, and review variability between criteria.

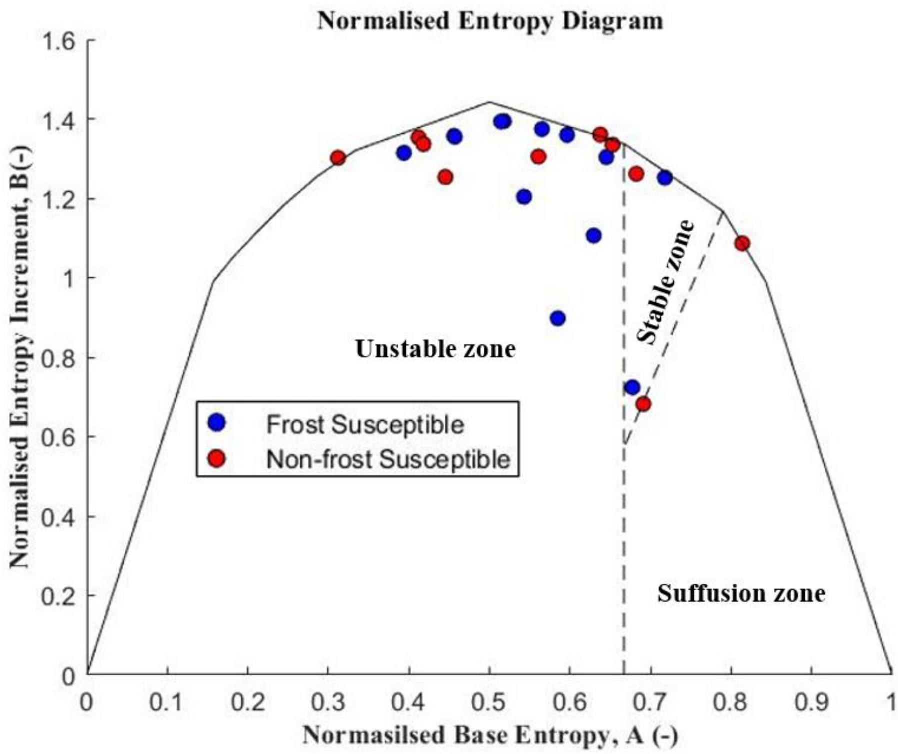

Figure 1. Normalised entropy diagram showing limits for susceptible and non-susceptible soils from Beskow, (1935), Corothors (1948), Croney (1949), Armstrong and Csathy (1963), Nielsen Rayschenberger (1952), Pietrzyk (1980), Jessberger (1969), Hartel (1967), and Riis (1948).

Observing the location of frost susceptible soils, they are predominantly grouped in the unstable zone. Although, both susceptible and non-susceptible soils are located here. This suggests that differences in approaches used when determining criteria may greatly influence the prediction of susceptibility. It can also be noted that non-susceptible soils are mostly located at the limits (i.e. A=0.3 and A=0.85) of the data. Larger fractions of both coarser and finer materials may indicate lower susceptibility**.** Finer fractions are known to be important in the development of frost heave, particularly when considering the degree filling in the voids. However, if the void space is dominated by fine particles soils have been shown to become less susceptible to frost action. This may also be the case for non-susceptible soils with larger A-coordinates, such as those in the stale zone. A greater B-coordinate indicates greater fraction variety and stronger particle fabric; hence this may limit the infiltration of water or prevent changes in the internal grain structure. However, it is difficult to draw any meaningful relationship between susceptibility and the criteria evaluated in this study. Figure 1 appears to support claims that PSD is not a useful indicator. Although, it may be suitable to state that Figure 1 highlights significant variability between PSD based criteria. This may be appropriate as many of the criteria assessed were determined using differing testing procedures, soil types, and freezing conditions. Therefore, Figure 1 aids in explaining the lack of consensus around PSD based criteria.

It is interesting to note that most soils in Figure 1 are found in the unstable zone. Whilst further investigation is needed, it may be speculated that during the thawing cycle phase changes contribute to losses in internal stability which may be affected primarily by density, rather than PSD. However, PSD is known to affect soil fabric. The expansion of ice lenses in the pore space during freezing has been shown to disrupt interparticle contacts, breaking the soil fabric structure (e.g. Dagesse 2010). During thaw, the lack of a stable fabric structure is prone to cause void collapse. Moreover, should this occur in soils with greater Fc, inefficient drainage conditions may induce greater, and perhaps prolonged, instability. The influence of freezing induced fragmentation may also be significant, and further contribute to unstable behaviours by increasing the Fc within soils. Therefore an alternative, or coupled, method of susceptibility assessment should include the influence of the PSD on the internal stability during and after the thawing process. Further research is therefore warranted.

EFFECTS OF PSD ON FROST HEAVE

Whilst it is difficult to establish a relationship between PSD and frost action in Figure 1, the effects of gradation on the development of frost heave may prove insightful. To do so, experimental data from Bilodeau et al. (2008) and Hao et al. (2023) are used.

Data from Bilodeau et al. (2008) was used to assess the impact of gradation on the frost heave susceptibility of 3 materials: Limestone, Basalt and Gneiss. The same PSD curves were used to represent the 3 material types and contained differing percentages of Fc, shown in Table 2:

Table 2. Soil ID's and their respective fines content percentages from Bilodeau et al. (2008)

Soil ID	Fines content (%)
F	11.7
M	9.5
C	5.7
U	3.6
WG	15.2
FM	11.7

Tests were conducted under conditions in a freezing cell at a temperature of -4°C at the bottom of the cell and 2°C at the top. Of those assessed, it was deemed that soils: **U** and **C** were deemed non-susceptible, whereas the other soils were deemed 'low frost-susceptible' by Chamberlin's criteria (Chamberlain 1981). Whereas soils **M**, **C** and **U** are classified as non-susceptible using the Norwegian criteria (Saetersdal, 1981). Considering the Gneiss material, all the gradation curves would be considered non-susceptible under both criteria. Therefore this data may be used to investigate the relationship between heave, PSD, and material type.

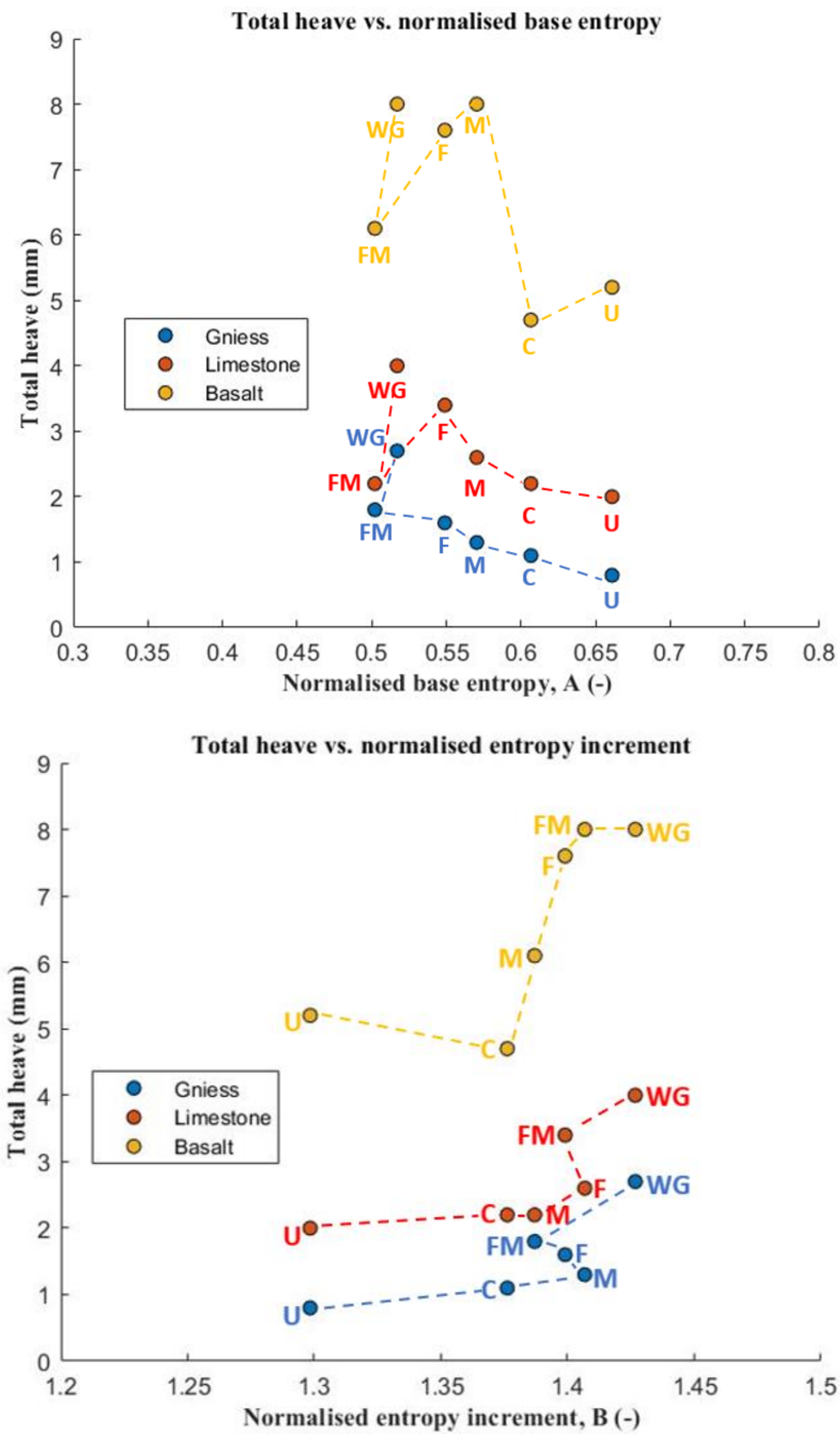

Figure 2. a) Total frost heave experienced in three soil types compared with the A-coordinate and b) the B-coordinate from Bilodeau et al. (2008).

Figure 2 a) and b) show the total frost heave (in mm) for the 3 materials. In Figure 2 a) the relationship between total heave and the A-coordinate is shown. In 2 a) there is an apparent relationship between the total heave experienced in the soils and their A-coordinates for all materials. As the A-coordinate decreases the total heave generally increases. This is likely due to the increase in finer material, which prevents drainage (i.e. ensuring there is water to freeze). For all materials, **U** and **C** have the largest A-coordinates and produce the lowest heave, suggesting that the criteria by Chamberlin may be accurate. This may also be extended to the Norwegian criteria as soil **M** is within close proximity to these soils. The greatest heave can be seen in soil **WG** in all materials, which also contains the greatest Fc percentage and the 2nd lowest A-coordinate. Regarding the different materials used, it is evident that there is a relationship between the frost heave and the material. It is interesting to note that whilst the Gneiss is deemed non-susceptible by both criteria, a similar heaving relationship is seen when compared to the Limestone and Basalt despite experiencing significantly less heave.

Observing Figure 2 b), the relationship between total heave and the B-coordinate (i.e. fraction variety) is seen for all materials. Soils **U**, **C** and **M** show the least heave and have the lowest B-coordinates in all materials. In particular, soil **U** is more distinct than its other non-susceptible counterparts. Soil **WG** shows the greatest total heave and has the largest B-coordinate. The figure suggests that as the finer material increases the heave also increases. All PSDs contained the same number of fractions; therefore the relationships indicate heave behaviour relative to the degree of filling within the soil fractions, something unable to be quantified using descriptors such as C_u. In fact, there seems to be a link between PSD, permeability and heave. Research by Feng et al. (2019) has shown that permeability may also be accurately predicted in terms of grading entropy coordinates, with a trend of permeability increase as the A-coordinate increases and the B-coordinate reduces.

To further investigate these relationships, data from Hao et al. (2023) is examined. This data was obtained via a different experimental procedure, it consists of 3 PSDs used to make 9 specimens with different levels of Fc (5% 10% and 15%) and tested at different hydraulic pressures (5kPa, 10kPa and 20kPa). The results were obtained via 1-D freezing tests. The coarser fraction consisted of China ISO standard sand and the finer fraction was a clayey-silt material. These soils were classified as a coarse sand according to the "Test Methods of Soils for Highway Engineering" JTG 3430–2020, are deemed non-frost heave susceptible. Tests were conducted at freezing temperatures -2.2°C, -4.4°C, -6.6°C. Figure 3 a) and b) show the total frost heave at all freezing temperatures with the normalised entropy coordinates.

In Figure 3 a) and b) it is evident that the same relationships established in the previous dataset are also present despite the difference in experimental procedure. Also note that despite these soils being deemed non-susceptible, all experienced frost heave. In Figure 3 a) as the A-coordinate decreases the total heave increases, further highlighting the effect of Fc on the development of ice lenses. Moreover, as the B-coordinate increases the total heave increases. Note that these PSDs also contained the same number of fractions, highlighting the effect of filling due to the fines within the soils (i.e. the relationship between PSD and permeability). The effect of freezing temperature and hydraulic pressure is also apparent, the greatest heave is experienced in soils with lowest temperature, greatest pressure and the smallest A-coordinates. Note however that these effects are independent of PSD. This is further shown in Figure 3 b), where heave increases with the addition of fines and a decrease in temperature.

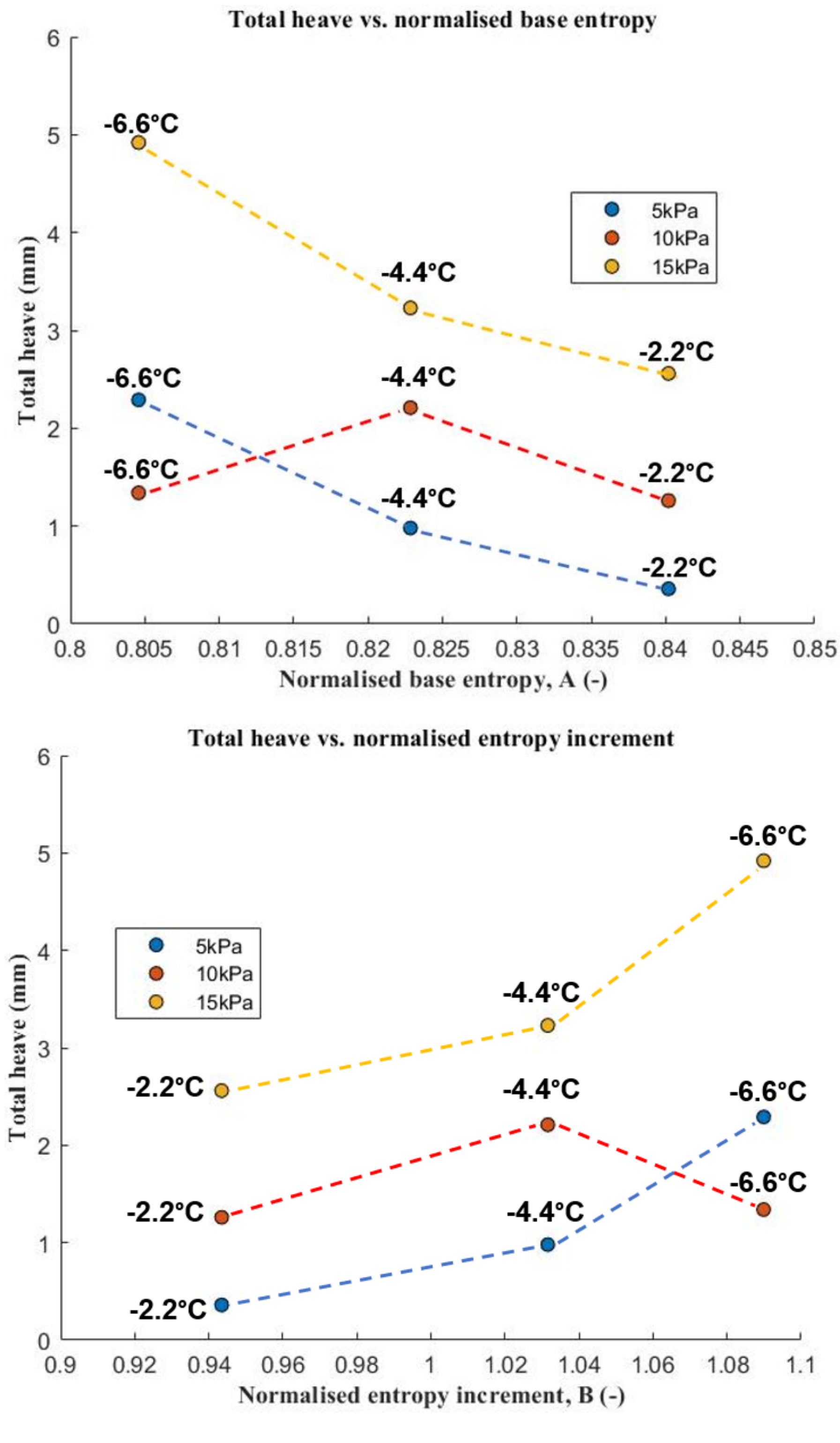

Figure 3. a) Total frost heave experienced in soils at different hydraulic pressures and freezing temperatures compared with the A-coordinate and b) the B-coordinate, from Hao et al. (2023). Note that increasing Fc is shown by decreasing A-coordinates and increasing B-coordinates.

DISCUSSION AND CONCLUSIONS

This work has investigated the effect of PSD on frost susceptibility and heave by reviewing a number of well-known PSD based criteria and experimental data from Bilodeau et al. (2008) and Hao et al. (2023). In this review, susceptibility was determined based on the criteria being assessed. On this basis, there was no clear distinction between susceptible and non-susceptible soils, suggesting that either PSD is not a useful indicator of frost susceptibility, or that the variety in proposed criteria has introduced significant variability in the outcome of frost susceptibility. The latter explanation may be more appropriate, as when analysing data from different experimental procedures, a relationship between PSD and total frost heave was evident. Further experimental study is needed if a meaningful PSD based criteria is to be established. An interesting observation was that many susceptible soils were located the unstable zone of the normalised entropy diagram, suggesting that instability as a result of density effects may be related to the PSD. Hence the stability criteria might be an effective indicator of susceptibility to infrastructure damage when considering thaw. However further investigation is needed. The key findings of this study are summerised below:

- Investigating a large number of PSD criteria revealed a lack of distinction between susceptible and non-susceptible soils. This may have implications on geotechnical design as the choice in criteria may provide highly variable outcomes on susceptibility.
- Despite the lack of agreement in the reviewed PSD criteria, a relationship between PSD and frost heave was observed over two experimental datasets. Soils which were deemed to be susceptible to heave also shared greater percentages of Fc. This was shared over both experimental data sets and seemed to confirm the relationship between PSD, permeability and heave.

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