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Minimum dry density in terms of grading entropy coordinates

E. Imre
EKIK HBM Research Centers and Bánki Donát Faculty of Mechanical and Safety Engineering, Óbuda University, Budapest, Hungary

D. Barreto
Edinburgh Napier University, UK

M. Datcheva
Institute of Mechanics and Institute of Information and Communication Technologies, Bulgarian Academy of Sciences, Sofia, Bulgaria

V.P. Singh
Texas A & M University, USA

W. Baille
Ruhr-Universität Bochum, Germany

S. Feng
Birmingham City University, UK
University of Bristol, UK

T. Firgi
Ybl Miklós Faculty of Architecture and Civil Engineering, Óbuda University, Budapest, Hungary

ABSTRACT: In this paper, the grading entropy method is applied together with advanced interpolation methods in order to establish empirical relationships between grading entropy coordinates and minimum dry density of sands. Three databases consisting of 94 samples (in total) of artificial mixtures of natural sands with fractal or continuous grain size distributions were used to evaluate three different types of empirical relationship. Whilst some databases have been previously published, recently acquired data has also been considered. The results show that there is a strong relationship between grading entropy coordinates and the minimum dry density.

1 INTRODUCTION

The experimental works by Kabai (1968, 1972, 1974) in relation to the dry density of sands from the Danube river concluded that the ratio of the minimum to the maximum dry density was basically constant, although it decreased as the soil became slightly plastic. Lőrincz (1986) also observed that a mixture of spheres with various radii can be denser if the radii values are selected from a larger size interval. Consequently, the dry density of the soil increases with the maximum particle diameter d_{max}. Furthermore, if the number of size fractions (N) in a sand mixture increases density is also expected to increase. Lőrincz (1986) tested both fractal and gap-graded grain size distribution with fixed N between 1 and 5 using grading entropy coordinates and found that the minimum dry density had a minimum value at around $A = 2/3$ (A is the relative base entropy, one of the grading entropy coordinates). This minimum value

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occurred for gap-graded mixtures. The aim of the present study is to extend the findings of these pioneering works with new experimental work (referred to as the new Óbuda-Bochum database) as well as other theoretical work by Imre et al (2009, 2011, 2013 and 2017). A key aspect is that a relationship between grain size distribution and dry density cannot be generally determined empirically due to the large number of possible grading curves hence an interpolation technique is used (Imre et al, 2009, 2017) to assess the success of various statistically derived PSD-density relationships in terms of grading entropy coordinates. The grading entropy concept is described in detail by Lőrincz (1986) and characterises any grading curve using an abstract system of N fractions. Each fraction (which may represent the weight of material collected in a given sieve during an experimental test) has a diameter (d) range that doubles in size (e.g. $d_0 \le d < 2d_0$, …, $2^{22}d_0 \le d < 2^{23}d_0$, $2^{23}d_0 \le d < 2^{24}d_0$, where d_0 may be 2^{-22} mm, the size of a silica tetrahedron). The relative frequencies of the fractions fulfil that:

$$\sum_{i=1}^{N} x_i = 1, x_i \ge 0, N \ge 1 \tag{1}$$

where the integer variable N is the number of the fractions between the finest and coarsest non-zero fractions, and x_i is the relative frequencies of fraction i (i.e. it may be the mass percentage retained in each sieve) The grading entropy can be separated into the base entropy (S_0) and the entropy increment (ΔS), defined as follows:

$$S_0 = \sum x_i S_{0i} = \sum x_i i \text{ and } \Delta S = \frac{-1}{\ln 2}\sum_{x_i \ne 0} x_i \ln x_i \tag{2}$$

where S_{0i} is the i^{th} fraction entropy (=i). The normalised forms of S_0 and ΔS are the relative base entropy (A) and the normalised entropy increment (B), respectively:

$$A = \frac{(S_0 - S_{0min})}{(S_{0max} - S_{0min})} \text{ and } B = \frac{\Delta S}{\ln N} \tag{3}$$

where S_{0max} and S_{0min} are the entropies of the largest and smallest fractions, respectively. Any grading curve can be represented as a point in terms of the non-normalised or normalised entropy coordinates. The inverse image of the maximum line of the diagram (critical values of the map) is the optimal line. The optimal point or grading curve has finite fractal distribution with the following relative frequencies:

$$x_1 = \frac{1}{\sum_{j=1}^{N} a^{j-1}} = \frac{1-a}{1-a^N},\ x_j = x_1 a^{j-1} \tag{4}$$

where a is the root of equation (5) as defined by Imre, et al (2009):

$$y = \sum_{j=1}^{N} a^{j-1}[j - 1 - A(N-1)] = 0 \tag{5}$$

The optimal grading curve with finite fractal distribution is a kind of mean grading curve of the grading curves for a given A. The inverse image of a regular value of the diagram with a given A and B ordinate is an "N-3 dimensional sphere" in the N-1 dimensional, closed simplex. The mean grading is the optimal one related to the given A.

2 MEASUREMENTS AND DATABASE - MINIMUM DRY DENSITY

The soils and testing methods are described in Imre, et al (2009, 2013, 2017). For this preliminary study three different databases were used. They are here referred to as the (i) Lőrincz, (ii) Kabai and (iii) Óbuda-Bochum databases, respectively. For the present analysis 50, 23 and 21 data samples were used from the databases (i), (ii), and (iii), respectively. In all databases, artificial mixtures of sand with different gradations were tested for dry density according to DIN 18126, and using various sample sizes. Values of the minimum solid volume ratio (s_{min}) were evaluated in terms of these dry density tests. The rationale for using such databases is as follows:

The Lőrincz (1986) database includes mixtures with optimal gradings with particle sizes varying between 0.07 and 4 mm with different values of relative base entropy (A); the Kabai (1972) database includes continuous data considering a slightly different value of d_0; the Óbuda-Bochum database (with data produced from ongoing research) includes unpublished results consisting of optimal gradings with A = 2/3. In summary, the databases have been chosen/developed in order to consider a wide range of particle sizes and distributions.

3 RESULTS AND ANALYSIS

In order to assess the effectiveness of using the grading entropy coordinates to estimate the minimum solid volume ratio (s_{min}), three types of empirical equation were tested on the basis of the experimental data. These are as follows:

$$s_{min} = C_3 S_0^{C_1} \Delta S^{C_2} \tag{6}$$

$$s_{min} = C_3 S_0^{C_1} (\Delta S + 1)^{C_2} \tag{7}$$

$$s_{min} = C_1 S_0 + C_2 \Delta S + C_3 \tag{8}$$

According to the results shown in Table 1 and Figure 1, the fitting parameters for each of the equations used were similar and it may be argued that additional statistical tests are required. The evaluation of these relationships must also be made within the context of their range of validity (which differs for each of the databases used). Note that despite its relative simplicity, Equation 8 provided the best fit for all databases. Similarly, amongst the three databases used, the Lőrincz database produced the best R^2 values.

4 SUMMARY AND CONCLUSIONS

The grading entropy method was applied using some experiments made on artificial mixtures of natural sands with fractal or continuous grain size distribution to determine a relationship between grading curves and dry density values in terms of solid volume ratio. Results indicated that there is a strong relationship between grading entropy coordinates and density. Despite its simplicity a fully linear relationship provided the best fit. Results however are dependent on the database used and must also be considered within the context of their range of validity.

A linear relation was found for the tested fractal distributions or continuous mixtures which supports the idea raised in Edwards' Statistical Mechanics approach that the dry density and the entropy are closely related. Therefore, by means of further research, a link between minimum solid volume ratio and the statistical mechanics approach (Baule et al. 2016) may be established. Further research is also suggested on gap-graded mixtures

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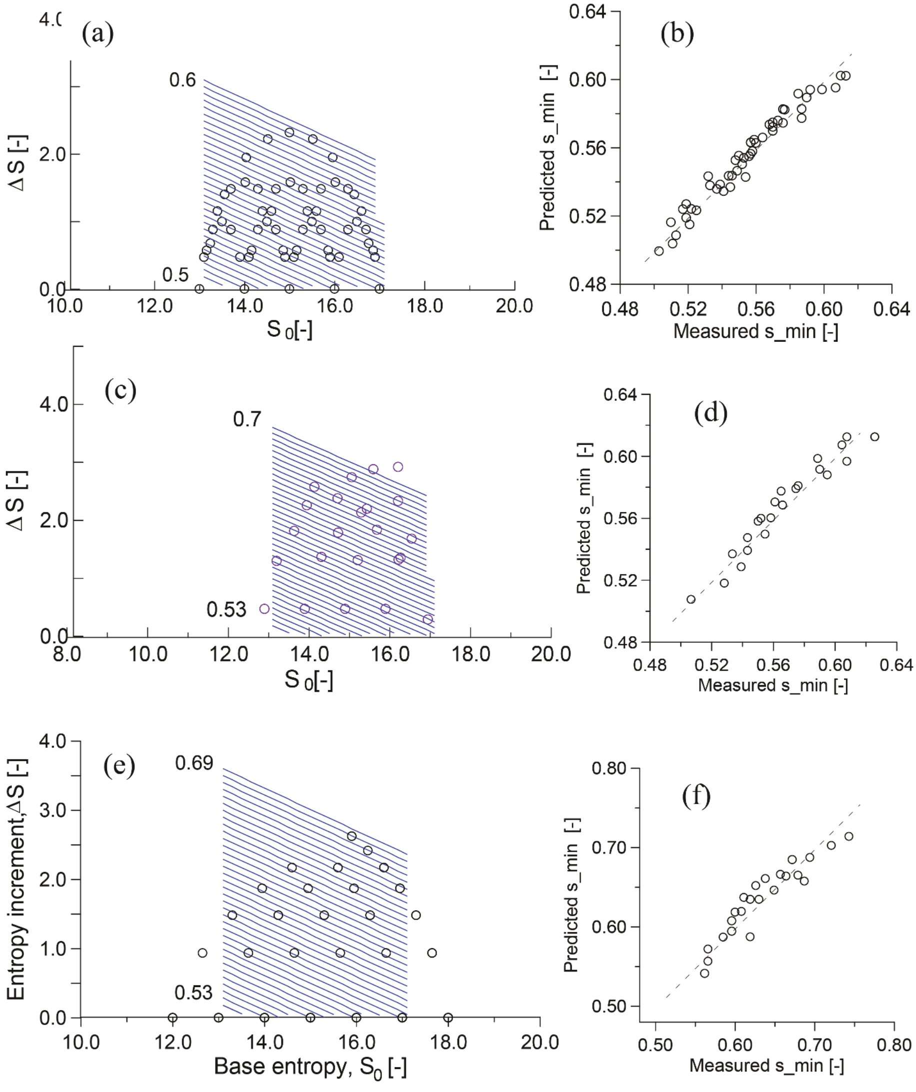

Figure 1. (s_{min}) isolines shown with gradings of experiments using on the non-normalised entropy diagram (left) and scatter plots for Equation. (8) (right.) for databases (a, b) Lőrincz; (c, d) Kabai and (e, f) Óbuda-Bochum.

Table 1. Fitting parameters and R^2 values for each of the empirical relationships and databases.

	Equation 6				Equation 7				Equation 8			
Database	C_1	C_2	C_3	R^2	C_1	C_2	C_3	R^2	C_1	C_2	C_3	R^2
Lőrincz	0.46	0.06	0.17	0.92	0.53	0.12	0.12	0.94	0.02	0.03	0.23	0.96
Kabai	0.41	0.08	0.21	0.76	0.38	0.18	0.19	0.82	0.02	0.05	0.32	0.87
Óbuda-Bochum	0.32	0.11	0.22	0.90	0.32	0.18	0.20	0.92	0.01	0.04	0.33	0.94

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