THE SATURATED HYDRAULIC CONDUCTIVITY OF 2-FRACTION GRANULAR SOILS AND THE INTERNAL STABILITY

Conference Paper · November 2020

8 authors, including:


Agnes Balint
Óbudai Egyetem


Daniel Barreto
Edinburgh Napier University

Adildorj Khaliunaa
Óbudai Egyetem

Tsendsuren Boldbaatar
Óbudai Egyetem

THE SATURATED HYDRAULIC CONDUCTIVITY OF 2-FRACTION GRANULAR SOILS AND THE INTERNAL STABILITY

Emőke Imre[1*], Zsombor Illés[2], Ágnes Bálint[1,3], Daniel Barreto[4], Adildorj Khaliunaa[3], Boldbaatar Tsendsuren[3], Dang Thi Quynh Huong[3], Lamas Lopez Lizeth Guadalupe[3]

[1] Hydro-Bio-Mechanical Systems Research Center, EKIK Óbuda University, Budapest, Hungary, e-mail: imre.emoke@kvk.uni-obuda.hu, *mobile: +36202892656; [2]Department of Engineering Geology and Geotechnics, Budapest University of Technology and Economics, Budapest, Hungary, e-mail:* zsombor.illes@epito.bme.hu, *phone: +36 1 463 3004; [3]Institute of Environmental Engineering and Natural Sciences, Óbuda University, Budapest, Hungary, e-mail: balint.agnes@uni-obuda.hu, mobile: +36303721342, [4]Edinburgh Napier University, U.K., e-mail:D.Barreto@napier.ac.uk,*

Abstract *The four grading entropy coordinates can be used for soil classification on the basis of grain size and the grading curve shape (similarly e.g., to the diameter values). The grading entropy coordinates give information on several basic Physics features of soil like minimum dry density, internal structure and degree of degradation for natural soils. Therefore, assumingly, using them, good permeability k regression relations can be elaborated. In this work some laboratory tests are made for saturated permeability on fractally distributed sand mixtures (which are mean grading curves with predetermined composition). After the first stage of the measurement it is found that, the results indicate that the preciseness is better if only the data of non-segregating, internally stable mixtures are used, indicating the importance of selecting non-segregating mixtures in laboratory tests.*

Keywords: *granular matter, saturated water permeability, grading curve, grading entropy, segregation.*

INTRODUCTION

The permeability regression with both the well-established (d_{10}, void ratio (e) and Kozeny term e3/(1+ e) [1 to 3]) and the new (grading entropy type) variables of the grading curves are examined in the research, the grading entropy variables are included and combined with the well accepted variables. The entropy coordinates give information on minimum dry density, internal stability and degree of degradation for natural soils. Using together them with usual grading curve parameters to elaborate permeability regression form is physically acceptable since the grading entropy coordinates can be interchanged with mean log diameter, can be amended by d_{10} and relative density information which are missing from the grading entropy coordinates.
In this part of the work, the results of some laboratory permeability tests are presented, which were made at the Laboratory of the Engineering Geology and Geotechnics Department of the Budapest University of Technology and Economics.

For the measurement 3 series of - artificial soil mixtures of natural grains with 2 neighbor fractions were used. The measured data were analyzed in terms of typical *k*- d_{10} relationship. The main result of the study was that the *k*- d_{10} relationship had different slope for A<2/3 and A>2/3 within each series. It can be noted that the natural granular soils has generally A>2/3 since only these have stable internal structure. Connecting these data for all series, the result was a nice quasi-linear relationship. Some of the results are presented here.

Grading entropy

The statistical entropy (the entropy of a distribution function) is presented in many textbooks and can be formulated as follows in the discrete case. Let us consider *M* elements in *m* equal cells, M_i is the number of the elements in the *i*-th cell. The statistical entropy S_s is:

$$S_s = Ms \tag{1}$$

where *s* is the specific entropy, or the entropy of an element given by

$$s = -\sum_{i=1}^{m} \alpha_i \log_b \alpha_i \tag{2}$$

In equation (2), *b* is the base of the logarithm, and α_i is the relative frequency of the i-th cell, given by

$$\alpha_i = \frac{M_i}{M} \tag{3}$$

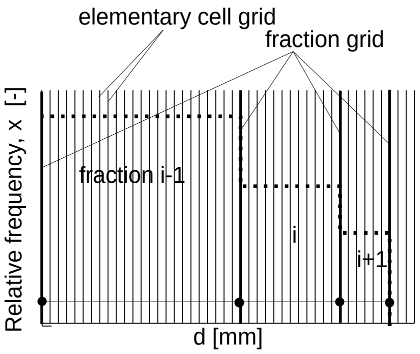


Figure (1) The grading density curve embedded in the elementary cell grid, assuming uniform distribution within the fractions.

For the statistical entropy [4-8] of a finite discrete grain size distribution, a uniform cell system is used. In the case of the empirical grading curve, the base of the logarithm is set to 2 in the statistical entropy formula:

$$s = -\frac{1}{\ln 2} \sum_{i=1}^{m} \alpha_i \ln \alpha_i \tag{4}$$

So, that the maximal value of the specific entropy of a two-cell system could be equal to 1 where the relative frequencies of a two cells are equal.

The empirical grain size distribution curve is considered as a finite discrete distribution. The statistical entropy is computed using two statistical cell systems (Figure 1). The so called fractions which are measured - are defined by successive multiplication with a factor of 2, starting from an arbitrary d_0 as follows (*j* =1, 2.., Table 1).

$$2^j d_0 \geq d > 2^{j-1} d_0 \tag{5}$$

where fractions are numbered by *j* (serial number). The elementary cells are with d_0 width assuming that the distribution within a fraction is uniform (Figure 1). The "smallest diameter" d_o may be practically taken as equal to the height of SiO_4 tetrahedron ($d_o=2^{-26}$ m, ~2,68E-8m).

Table (1) Definition and properties of fraction

j	1	23	24
Limits in d_0	1 to 2	2^{22} to 2^{23}	2^{23} to 2^{24}
S_{0j} [-]	1	23	24

The number of the elementary cells C_i in the fraction *i* is equal to:

$$C_i = \frac{2^i d_0 - 2^{i-1} d_0}{d_0} = 2^{i-1} \quad (6)$$

The relative frequency of any elementary cell in fraction *i* is equal to:

$$\alpha_i = \frac{x_i}{C_i} \quad (7)$$

where x_i is the relative frequency of fraction *i*.
The grading entropy *S* is derived by using secondary cells and inserting the relative frequency of the secondary cell α_i:

$$S = -\frac{1}{\ln 2} \sum_{x_i \neq 0} C_i \frac{x_i}{C_i} \ln \frac{x_i}{C_i}, x_i \geq 0 \quad (8)$$

where C_i is the number of the elementary cells in fraction *i*, and x_i is the relative frequency of fraction *i*. The grading entropy *S* is split into the base entropy S_o and the entropy increment ΔS:

$$S = S_0 + \Delta S \quad (9)$$

The base entropy S_0 and the normalized form *A*:

$$S_0 = \sum x_i S_{0i} = \sum x_i i \quad (1)$$

$$A = \frac{S_o - S_{o\min}}{S_{o\max} - S_{o\min}} \quad (2)$$

where S_{0k} is the *k*-th fraction entropy (Table 1), which is defined as follows (Table 1):

$$S_{ok} = \frac{\ln C_k}{\ln 2} \quad S_{ok} = k \quad (12)$$

S_{0max} and S_{0min} are the entropies of largest and smallest fractions, respectively. The entropy increment ΔS and the normalized version *B*:

$$\Delta S = -\frac{1}{\ln 2} \sum_{x_i \neq 0} x_i \ln x_i . \quad (3)$$

$$B = \frac{\Delta S}{\ln N} \quad (4)$$

where S_{0i} is the grading entropy of the *k*-th fraction. The grading entropy parameters induces a secondary structure on the space of the grading curves. The A = const. condition defines parallel *N*-2 dimensional hyper-plane sections of the *N*-1 dimensional simplex, the A = const., B=const. condition defines *N*-3 dimensional topological circles).
The relative frequencies of the fractions x_i (i = 1, 2, ..*N*) for each grading curve fulfil:

$$\sum_{i=1}^{N} x_i = 1, \qquad x_i \geq 0, \quad N \geq 1. \quad (5)$$

where *N* is the number of the fractions between the finest and coarsest non-zero fractions:

$$N = j_{\max} - j_{\min} + 1 \quad (6)$$

The relative frequencies x_i - and the space of grading curves with *N* fractions - can be identified with the barycentre coordinates in an *N*-1 dimensional simplex (see Figures 2-3).
Four maps can be defined between a grading curve space (i.e. *N*-1 dimensional, open simplex) and the two dimensional space of the entropy coordinates: the non-normalized $\Delta \rightarrow [S_0, \Delta S]$; normalized $\Delta \rightarrow [A, B]$; partly normalized $\Delta \rightarrow [A, \Delta S]$ or $\Delta \rightarrow [S_0, B]$. The map is continuous on the open simplex and can continuously be extended to the closed simplex. The diagram is symmetric and compact, having a minimum and a maximum boundary line (see Figures 4-7).
The relative base entropy *A* indicates the relative distance of the mean diameter from the maximum-minimum $\log_2$ diameter values. If A > 2/3 then enough large grains are present in a mixture to form gradually a skeleton and a stable soil matrix. The coarse particles “float” in the matrix of the fines if A < 2/3.
The internal stability criterion (Figure 8) for elongated grading curves includes a transitional zone, its boundary connects the maximum entropy points with fraction numbers less than *N*. Considering the fractal gradings, the soil is stable if $n < 2$, transitional between *n* at *A*=2/3 (*n* is varying in the function of *N*). The segregation criterion is similarly given in terms of *A* (Figure 8).

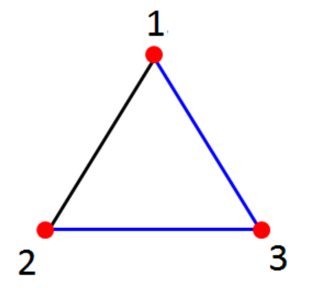


Figure (2) Standard simplex image with dimension 2.

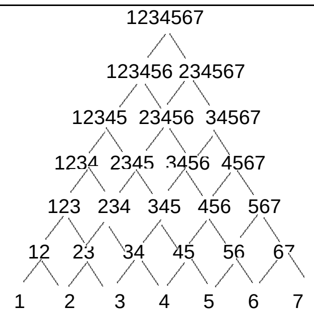


Figure (3) The lattice of the continuous sub- simplexes of the 6-dimensional simplex

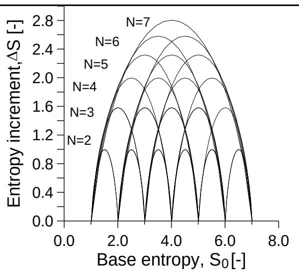


Figure (4) The maximum lines in the non-normalised entropy diagram of a simplex with N=7

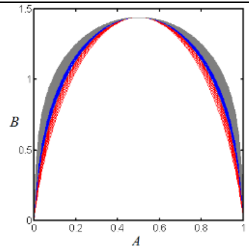


Figure (5) The maximum lines in the normalised entropy diagram of a simplex with N=7

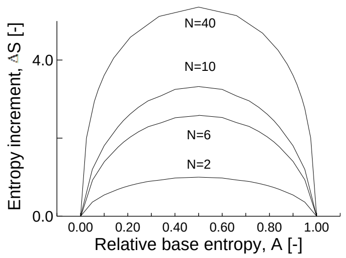


Figure (6) The maximum lines in the partly normalized diagram, using A, a simplex with N=7.

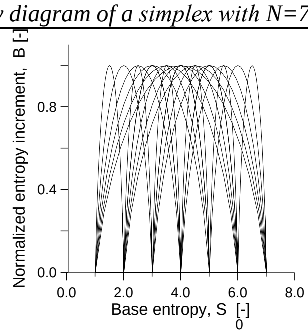
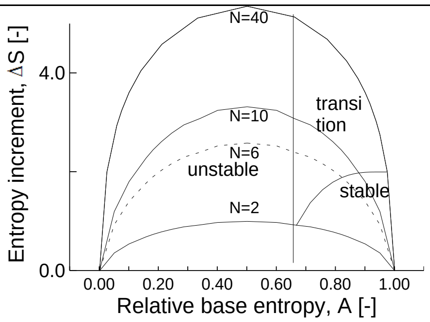
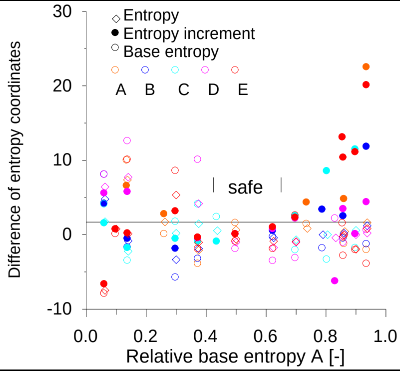


Figure (7) The maximum lines in the partly normalized diagram, using B, a simplex with N=7.



(a)



(b)

Figure (8) (a) Internal stability criterion and (b) segregation criterion of Lőrincz [4]

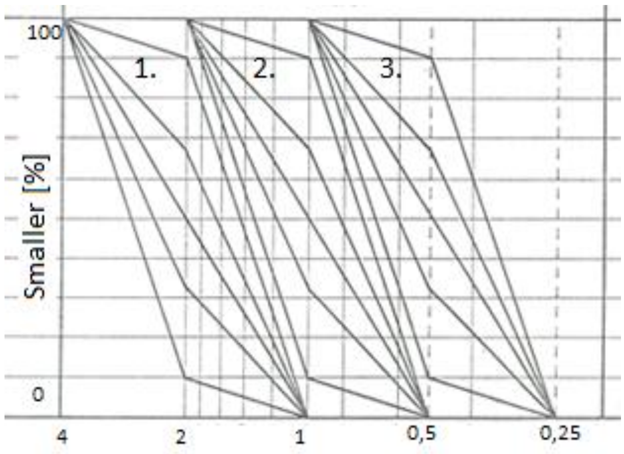


Figure (9) The earlier tested 2-fraction, continuous grading curve series 1, 2 and 3.

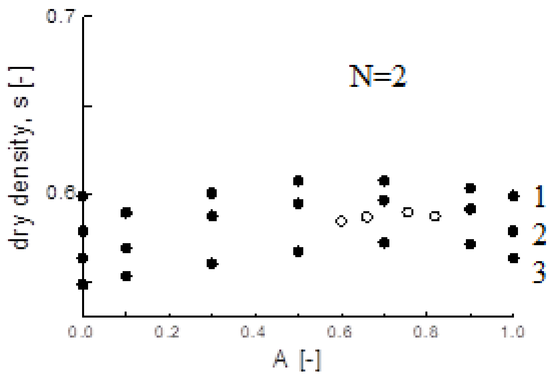


Figure (10) Measured minimum dry density in terms of A (open symbols indicates some repeats in the frame of an ongoing research).

According to some earlier test results, the entropy coordinates give information on minimum dry density, critical state friction angle, internal stability and degree of degradation for natural soils [4-10]. For example, using artificial soil mixtures of natural grains with 2 neighbor fraction grading curves, minimum dry density emax or smin test were made. According to Figures 9 to 10, it was found that the maximum was dependent on A for each soil series in the same way, encountered at around A = 2/3.

MATERIALS AND METHODS

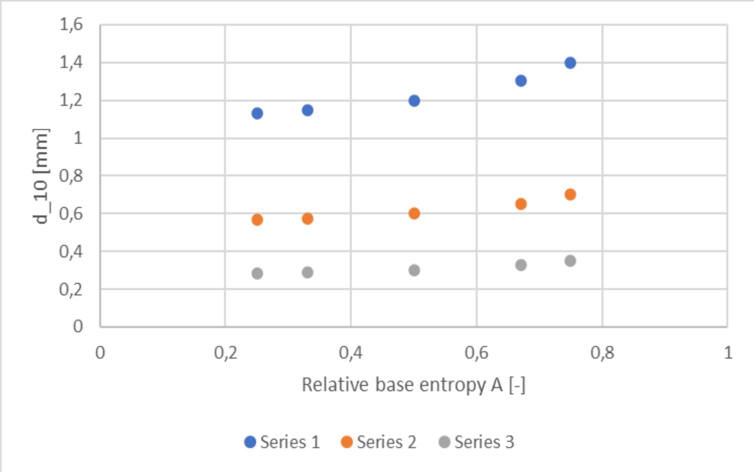
The 3 series of N=2 neighbor fraction mixtures used to measure the saturated water permeability at 5 different A parameter values: 0.25, 0.33, 0.5, ,0.67, 0,75. The four fractions (N =1) were: 0.25-0.5 mm, 0.5-1 mm, 1-2 mm 2-4 mm, (uniform distribution was assumed within the limits). The 2-fraction soil mixtures (Table 2) were tested with constant head permeability tests, and s_{min} test.

RESULTS

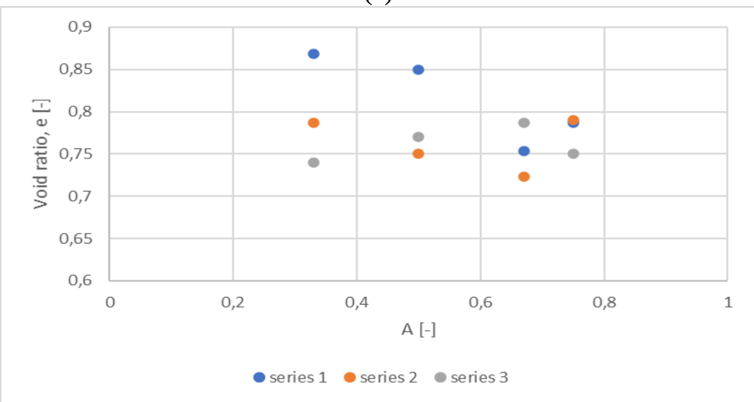
Results are shown in Figures 11 to 14. According to the results, for the tested gradings, the d_{10} has a monotonic increasing relationship with the relative base entropy A - the normalized mean log diameter, as expected. According to Figure 11, the maximum value of s_{min} of optimal series was at around A=2/3, the s_{min} for mixtures increased with mean diameter. Similarly, the k has a monotonic increasing relationship with the relative base entropy A. However, the $d_{10} – k$ relationship is not independent of the value of the relative base entropy A, which is neglected in the well-known relationships ([1] to [3]). The relationship has different slope for A<2/3 and A>2/3 within each series. It can be noted that the natural granular soils has generally A>2/3.

Table (2) The soils tested

Source of soil	No. of k Measurements	No. Grading Curves	Range d_{10} (mm)	Range k_{20oC} (cm/s)	Range C_U	Range C_C	N fraction number
Kvarc-Ásvány Kft. 8256 Ábrahámhegy-Kisörs	15	15	0.28 to 1.4	0.079-1.382	1.6 to 2.2	0.9 to 1.1	2



(a)



(b)

Figure (11) The characteristic d_{10} (a) and void ratio e (b) in terms of A for tested grading curve series 1, 2 and 3. The void ratio e is minimal at A=2/3.

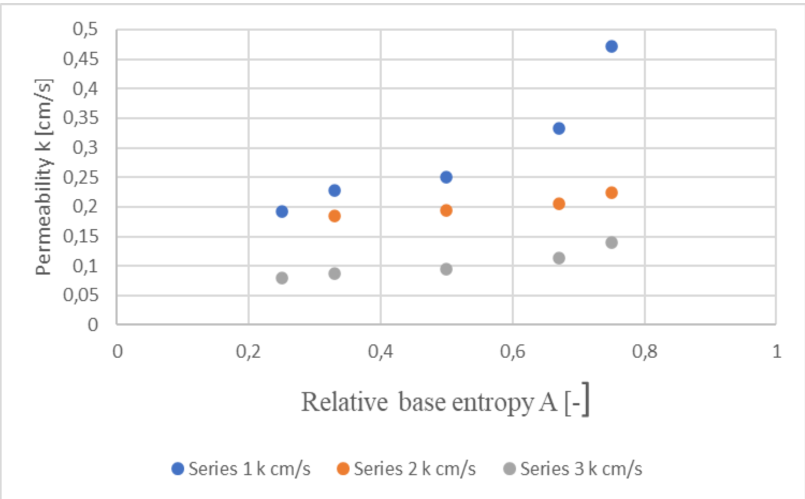


Figure (12) The measured permeability k in terms of A for tested grading curve series 1, 2 and 3.

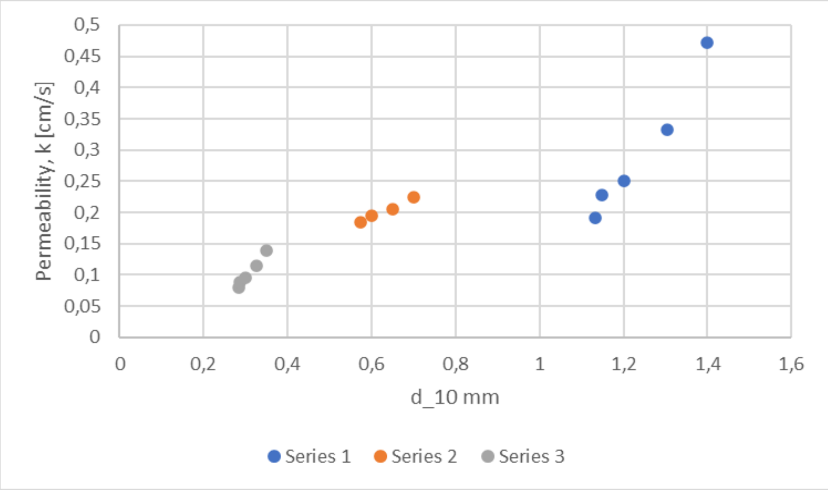


Figure (13) The measured permeability k in terms of d10 for tested grading curve series 1, 2 and 3.

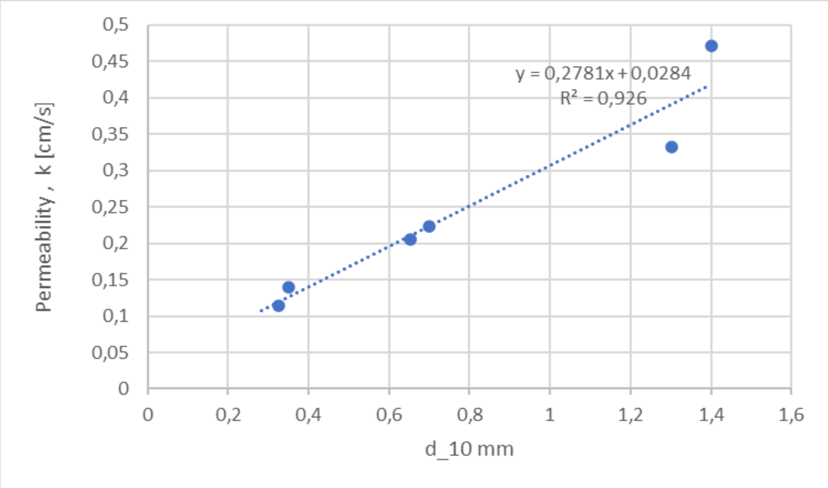


Figure (14) The measured permeability k in terms of d10. for A>2/3.

DISCUSSION AND CONCLUSIONS

The 3 series of N=2 fraction mixtures used to study the relation of saturated water permeability and the grading curves were selected at 5 different A parameter values: 0.25, 0.33, 0.5, 0.67 and 0.75. The result was as follows.

1. The d_{10} – k relationship follows the expectations with the following comment. It is not independent of the value of the relative base entropy *A*. The permeability k versus d10 relation seems to be is different for internally stable soils than for internally unstable soils.
2. The maximum value of s_{min} of optimal series was at around *A*=2/3, the s_{min} for mixtures increased with mean diameter. The tendency was not distinct for A<2/3.

According to the segregation rule (0.4<A<0.8), the first two A values are segregating mixtures. The internal stability is not ensured in the first three A (A<2/3) values. It can be assumed that the segregation during sample preparation causes the difference, the grading curve of the sample is likely not homogeneous is the sample is not internally stable. In other words, this can probably partly be explained by the fact that the internally stable soils are less sensitive to segregation during sample preparation.

The conclusion is as follows. Both the d10 (or A) – k relationship and – s_{min} – A relationship the expectations but is not independent of the value of the relative base entropy *A*, being different for internally stable soils and internally unstable soils. It can be assumed that the segregation during sample preparation causes the difference.

Acknowledgements

Thanks to the Kvarc-Ásvány Kft. 8256 Ábrahámhegy-Kisörs provided the sand. Thanks for the great support of dr. Rita Bodáné Kendrovics the Director of Institute of Environmental Engineering and Natural Sciences, Óbuda University; dr. Zoltán Tompai, Prof. Ákos Török, the Department of Engineering Geology and Geotechnics, Budapest University of Technology and Economics, and the rector of BME. The research is done in the form of a joint research of Óbuda University, Budapest, Hungary; Budapest University of Technology and Economics, Budapest, Hungary; Edinburgh Napier University, U.K; Università di Roma Tor Vergata, Italy; Politecnico di Milano, Italy; Ruhr-Universität Bochum, Germany; University of Bristol, U.K.

References

[1] Kozeny J. Über kapillare Leitung des Wassers im Boden. Akademie der Wissenschaften Wien, 1927, 136(2a): 271–306.

[2] Hazen A. Discussion of Dams on sand foundations. A. C. Koenig ed., Transactions of the American Society of Civil Engineers. 1911, 73(3): 199–203.

[3] Chapuis RP. Predicting the saturated hydraulic conductivity of sand and gravel using effective diameter and void ratio. 2004, CGJ. 41(5): 787–795. https://doi.org/10.1139/t04-022

[4] Lőrincz J. Grading entropy of soils. Doctoral Thesis. 1986. TU of Budapest. (in Hungarian).

[5] Lőrincz J. On granular filters with the help of grading entropy. Proc. of Conf. on Filters in Geotechnical and Hydr.Eng. Brauns, Heibaum. Schuler. BALKEMA. 1993. 67-69.

[7] Lőrincz J. On particle migration with the help of grading entropy. Proc. of Conf. on Filters in Geotechnical and Hydr.Eng. Brauns. Heibaum. Schuler. BALKEMA. 1993, 63-65.

[8] Lőrincz, J. "Relationship between grading entropy and dry bulk density of granular soils". Periodica Polytechnica, 34(3), pp. 255-265, 1996.

[9] Imre, E., Lőrincz, J., Szendefy, J., Trang, P., Nagy, L., Singh, V., Fityus, S. "Case studies and benchmark examples for the use of grading entropy in geotechnics", Entropy14(12), pp. 1079–1102, 2012. https://doi.org/10.3390/e14061079.

[10] Imre, E; Lorincz, J.; Trang, P. Q.; Csonka, I.; Kaczvinszki-Szabó, V. ; Telekes, G.; Goudarzy, M.; Rahemi, N.; Baille, W.; Schanz, T. Preliminary study on the relationship between dry density of sands and the grading entropy parameters In: 9 XVII European Conference on Soil Mechanics and Geotechnical Engineering - Reykjavík, Iceland : 1–6 of September 2019 : Geotechnical Engineering, foundation of the future : Conference proceedings Reykjavík, Izland : Icelandic Geotechnical Society, IGS, (2019) pp. 996-1002. , 7 p.