A szemeloszlás és a belső stabilitás kapcsolata

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Absztrakt: A szemeloszlási görbe empirikus eloszlási függvény, lépcsős függvény, rögzített statisztikai cellákkal. Diszkrét eloszlásnak tekintve, alkalmazva rá a relatív entrópia definícióját, levezethető két ún. entrópia-koordináta. Ezek jól osztályozzák az osztályozási görbéket, és statisztikailag jobbak információtartalom szempontjából, mint a jelenleg alkalmazott közelítő kvantilisek, vagy kvantilis típusú paraméterek hányadosai. Az elméleti és kísérleti munka elemezi a paraméterek fizikai tartalmát. Az eredményeket a következőképpen lehet összefoglalni. Az első entrópia-paraméter egy folytonos belső stabilitásmértéknek tűnik. A második entrópia-paraméter lehetővé teszi egy átlagos szemeloszlási görbe meghatározását véges fraktál szemcseméret-eloszlással az első paraméter minden egyes értékére vonatkozóan. A mikromechanikai eredmények azt mutatják, hogy az első paraméter hogyan kapcsolódik a belső szerkezethez. Matematikai eszközökkel az az eredmény nyerhető, hogy a belső stabilitási szabály szerinti stabil állapot valószínűsége nagyon alacsony, a természetben a stabil állapot fordul mégis elő, ugyanis a talajok degradációja determinisztikus. A belső stabilitás a mérnöki földszerkezetek kulcsfontosságú paramétere.

Kulcsszavak: kulcsszó1; kulcsszó2; kulcsszó3

Grading curves and internal stability

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Abstract: The measured grading curve is an empirical distribution function, a step function. This is considered here as a discrete distribution with fixed statistical cells. In the grading entropy theory it is characterized by the relative entropy resulting in two sets of entropy coordinates. These first and second grading entropy coordinates classify well the grading curves and are statistically more soundly based in terms of information content than the approximate quantile type parameters used at present. In the theoretical and experimental work on the grading entropy coordinates, the physical content of the parameters are analysed. The results can be summarized as follows. The first entropy parameter seems to be a continuous internal stability measure. The second one allows the definition of a unique, mean grading curve with finite fractal grain size distribution for fixed value of the first parameter. The first parameter is related to internal structure, proven here by DEM tools. It is shown by Math tools that the probability of a stable state of the grading entropy theory is very low. The generally occurring stable states in the nature are originated from the degradation which is deterministic. The internal stability of the engineering structures can be characterized by grading entropy.

Keywords: grading curve and grading entropy, internal stability, fractal, DEM

1. Introduction

The grading curves of soils contain a large amount of data. This can render its use in the evaluation of soil properties awkward. Hence, rules based on a few nominated particle diameters have been developed (i.e. coefficients of uniformity and curvature, c_u and c_c, respectively). In more developed cases, some parametric functions are fitted to the grading curves or other approaches are used. However, these approaches are not valid for gap-graded grain size distributions. It is shown here that the two grading entropy coordinate pairs (base entropy and entropy increment), proposed by Lőrincz ([1]) may characterize the grading curves more effectively. They contain all measured data in terms of statistical means, and are related to internal stability.

Table 1. Fractions

j [-]	1			23	24
Limits in d_o	1 to 2			2^{22} to 2^{23}	2^{23} to 2^{24}
S_{oj} [-]	1			23	24

2. Grading entropy

2.1. Space of grading curves

An abstract fraction system is defined. The diameter range for fraction j (j =1, 2...j, see Table 1) is:

$$2^{j} d_0 \geq d > 2^{j-1} d_0, \qquad (1)$$

where d_0 is the smallest diameter which may be equal to the height of the SiO_4 tetrahedron. The 2 base log of the diameter limits are integers, called abstract diameters. The relative frequencies of the fractions x_i (i = 1, 2, ..N) for each grading curve fulfil the following equation:

$$\sum_{i=1}^{N} x_i = 1, \quad x_i \geq 0, \quad N \geq 1. \qquad (2)$$

where the integer variable N –the number of the fractions between the finest and coarsest non-zero fractions– is used. The relative frequencies x_i can be identified with the barycentre coordinates of the points of an N-1 dimensional, closed simplex (which is the N-1 dimensional analogy of the triangle or tetrahedron, the 2 and 3 dimensional instances), and the space of grading curves with N fractions can be identified with an N-1 dimensional, closed simplex.

2.2. Grading entropy parameters

jThe grading entropy S is a statistical entropy, modified for the unequal cells (fractions are doubled). It can be separated into the sum of two parts [1]:

$$S = S_0 + \Delta S \qquad (3)$$

where S_0 is base entropy and □S is entropy increment. S_0 is a log "mean" of the diameter:

$$S_0 = \sum x_i S_{0i} = \sum x_i i \qquad (4)$$

where S_{0i} is the i-th fraction entropy (Table 1). The entropy increment:

$$\Delta S = -\frac{1}{\ln 2} \sum_{x_i \neq 0} x_i \ln x_i \qquad (5)$$

The relative base entropy A and normalized entropy increment B:

$$A := \frac{S_o - S_{o\min}}{S_{o\max} - S_{o\min}} = \frac{\sum_{i=1}^{N} x_i (S_{oi} - S_{o\min})}{N-1} = \frac{\sum_{i=1}^{N} x_i (i-1)}{N-1}, \quad B = \frac{\Delta S}{\ln N}. \qquad (6)$$

where S_{0max} and S_{0min} are the entropies of the largest and the smallest fractions, respectively.

Representing the space of the grading curves with N fractions by an N -1 dimensional, closed simplex, a secondary structure appear along to iso-surfaces of the normalised grading entropy parameters as follows.

The grading entropy parameter A is a linear function, the A = constant condition defines parallel hyper-plane sections of the N-1 dimensional simplex, which are disjunct subspaces in the space of the grading curves (Fig. 2). The grading entropy parameter B is a strictly concave function with a unique maximum for each A = constant value, which is a mean ("optimal") point:

$$x_1 = \frac{1}{\sum_{j=1}^{N} a^{j-1}} = \frac{1-a}{1-a^N}, \quad x_j = x_1\, a^{j-1} \qquad (7)$$

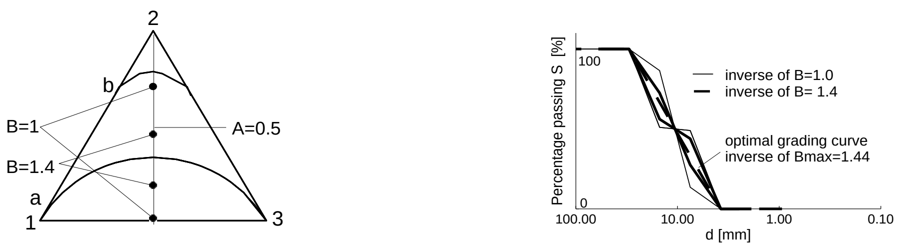
where a is the root of the following equation:

$$y = \sum_{j=1}^{N} a^{j-1}[j-1-A(N-1)] = 0 \qquad (8)$$

The optimal grading curves have fractal distribution, the fractal dimension is as follows [2,3].

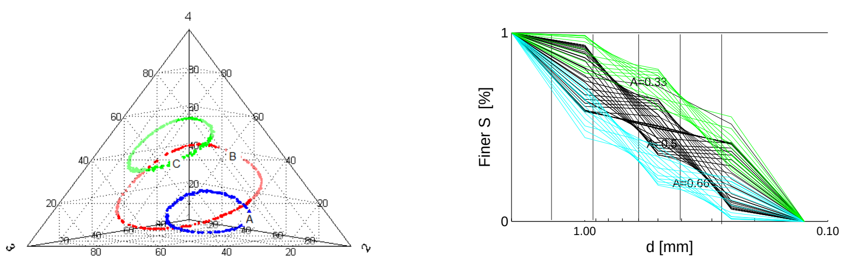
$$n = 3 - \frac{\log a}{\log 2} \qquad (9)$$

While a varies from 0 to 1 and 1 to ∞, A varies from 0 to 0.5 and from 0.5 to 1, n varies from ∞ to 3 and from 3 to -∞…respectively, at the two sides of the entropy diagram. Concerning the inverse image of a regular entropy diagram point [A,B] in the simplex is an N-3 dimensional sphere, "centered" to the optimal point (of the A = const, N-2 dimensional hyper-plane, Figs. 1-2).



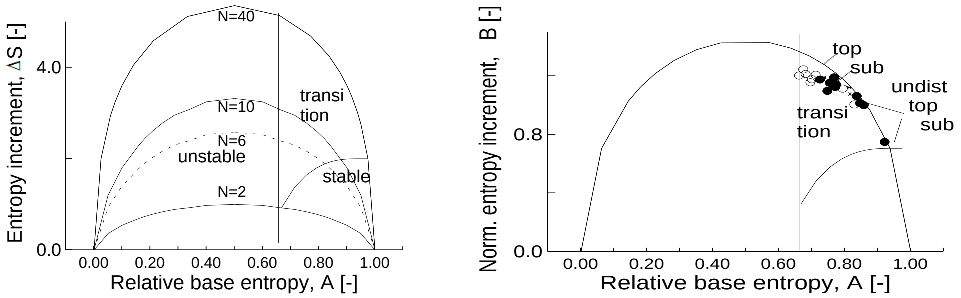
(a) (b)

Figure 1. N=3. (a) (b) The simplex points and the grading curves related to entropy parameters A=0.5 B=1. and A=0.5 B=1.4 which are topologically N-3=0 dimensional circles (point pairs and grading curve pairs).



(b)

Figure 2. N=4. (a) (b) The simplex points and the grading curves related to entropy parameters A=0.66 B=1.2, and A=0.5 B=1.2, and A=0.3 B=1.2, which are N-3=1 dimensional topological circles (in the simplex and in the space of the grading curves).



(a) (b)

Figure 3. (a) Internal or grain structure stability criterion in the non-normalized diagram. (b) Internal or grain structure stability criterion in the N=17 related normalized diagram, with the indication of the degradation example of waste rock in open pit mine rehabilitation (the topsoil samples are more degraded [10]).

2.3. Grading entropy to describe internal stability

Four maps can be defined between a grading curve space (N-1 dimensional, open simplex) and the two dimensional space of the entropy coordinates: the non-normalized $\Delta \rightarrow [S_0, \Delta S]$; normalized $\Delta \rightarrow [A, B]$; partly normalized $\Delta \rightarrow [A, \Delta S]$ or $\Delta \rightarrow [S_0, B]$. Maps are continuous on the open simplex and can continuously be extended to the closed simplex for fixed N. The internal stability rule of the grading entropy theory ([1]) is defined by vertical flow tests on a partly normalized entropy diagram shown in Fig. 3(a). There are three main zones. For A < 2/3, the mixtures are internally unstable, for A = 2/3 and A > 2/3 the soils are internally stable, the structure gradually builds up for elongated grading curves. Suffosion may occur in each zone. The rule can be interpreted such that in Zone I (if A < 2/3) the coarse particles "float" in the matrix of the fines and become destabilized when the fines are removed by piping. In the complementer zone (A = 2/3 and A < 2/3), the coarse particles form a skeleton, total erosion cannot occur. In Zone III, the structure of larger particles is assumingly inherently stable.

Table 2 The geometrical probability of the internally stable state in terms of the fraction number (volume of stable part A<2/3 and the total volume of the N-1 dimensional simplex)

N [-]	2	5	10	20	50
P(A>2/3)	3E-01	1E-01	4E-02	6E-03	2E-05

Table 3 Effect of grading entropy parameter A on the coordination number and critical friction angle - DEM and real experimental results

A [-]	Z [-]	Zm [-]	ϕ'_{crit} [^{o}]	$\phi'_{crit, real}$ [^{o}]

0.10	2.87	4.32	18.72	~33
0.33	2.34	4.41	18.43	~33
0.50	2.52	4.42	18.08	~34,5
0.66	2.83	4.47	17.85	~35.8
0.90	3.39	4.47	17.55	~36.5

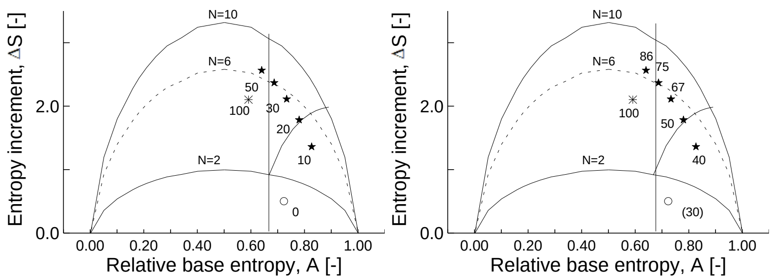
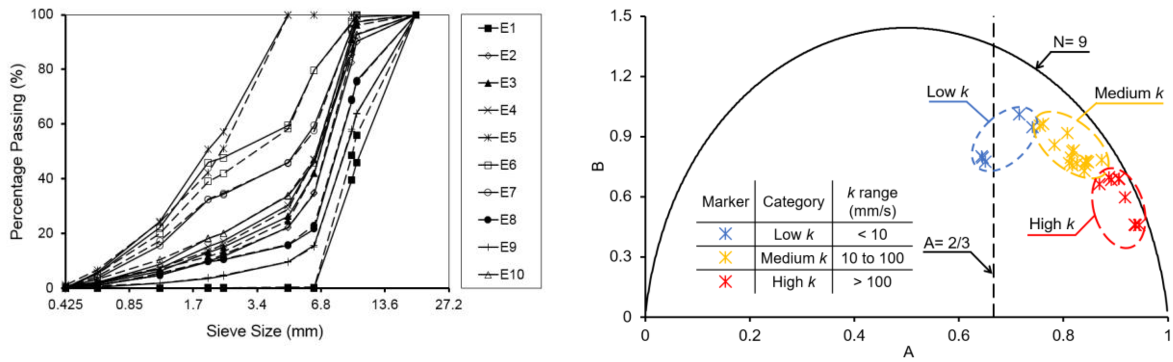


Figure 4. Liquefaction susceptibility of Houstun sand - fine mixtures, varying fine content. Tests of Negar Rahemi in partly normalized diagram. (a) Gradings with various fine content percentages. (b) Probability of sample liquefaction at the previous points. (Note: bracket means that only limited liquefaction may occur.)



(a)

(b)

Figure 5. (a) Grading curve set out of three tested sets with varying fine content. (b) Permeability variation in normalised diagram [11].

3. Analysis of the internal stability rule

3.1. Internal stability and probability

The optimal or fractal grading curves have fractal dimension between $-\infty < n < \infty$. The fractal soil is stable if $n < 2$, unstable at $A<2/3$ (n is varying in the function of N), transitional between these values. The natural soils [4 to 6] are generally fractal grading curves, the fractal dimension is between 2 and 3, being related to the stable-transitionally stable zones in terms of internal stability.

The geometrical probability expressed by the ratio of the volume of the simplex of the grading curves where A>2 /3 is met and the volume of the whole simplex tends to be zero (Table 2, Talata 2018 [7]). This contradiction can be explained by the fact that the degradation process is deterministic but its discussion is beyond the scope of this paper (see eg., [12]).

Soil degradation in mine rehabilitation through degradation of waste rock over short time period is measured by studying the grading curve data ([10]). The grading curves are plotted in the transitional stability zone on the entropy diagram, with near fractal dimensions 2.5 to 2.8. A distinct difference for all sub and top samples is found: the topsoil is more degraded (Fig 3b).

3.2. Insights from DEM simulations

Preliminary 3D DEM simulations of spherical particles using periodic boundaries were performed. Four fraction sizes were used: 0.125-0.25 mm, 0.25-0.5 mm, 0.5-1 mm, and 1-2 mm (uniform distribution was assumed within the limits). 2-fraction soils (N=2) were tested with various A values under drained triaxial conditions until the critical state was achieved.

These DEM specimens were isotropically consolidated to 200 kPa and subsequently sheared with an inter-particle friction coefficient of 0.3. As a result their initial density is close to minimum and the overall (Z) and mechanical (Z_m) coordination numbers reflect this, as shown in Table 3.

Note the coordination numbers represent the average number of contacts per particle for the specimen at the critical state. The mechanical coordination number is of particular relevance as it indicates the average number of contacts per particle, but in contrast to the overall coordination number, it only includes the particles which effectively transit stress (i.e. those that are part of strong force chains as normally reported in DEM studies).

Note that as *A* increases, Z_m also increases indicating that the number of particles effectively transmitting stress increases. Furthermore, as *A* increases, the difference between Z and Z_m decreases. This implies that as *A* increases the number of “rattlers” (i.e. particles that are not part of the strong force chains) reduces. In other words, as *A* increases, the specimens become inherently more stable. In contrast, lower values of A indicate a higher likelihood of “rattlers” -or fines- that are potentially erodible. Hence, there is a clear link between stability, base entropy and mechanical coordination number.

Furthermore, the critical angle of shearing resistance seems to be dependent of A. This dependence relates to the inherent stability of the strong force chains, but its discussion is beyond the scope of this paper. It must be noted that the range of variation is nevertheless limited.

3.3. Examples on the effect of fines

A series of 60 conventional triaxial compression tests are conducted on Hostun sand –silt mixtures to investigate the effects of fines on the undrained monotonic response of sand ([8]).

Fig. 4(a) shows the mixtures with various fine content, Fig. 4(b) demonstrates that transitionally stable mixtures can increasingly be prone to liquefaction during static loading with increasing fine content if precticallly all possible initial relative densities are considered([9]). In a similar study, the boundary between the "fines-in-sand" and "sand-in-fines" micro-structure, the threshold fines content is found at around *A*=2/3 ([9]).

A correlation between the normalised grading entropy coordinates and the coefficient of permeability is presented [11]. Permeability depends on the voids. Some permeability zones are identified on the normalised entropy diagram on the basis of k values measured on 3 sets of grading curves. These zones follow rule that with decreasing *A* the fine content is increasing and the porosity is decreasing (Fig. 5b).

4. Discussion

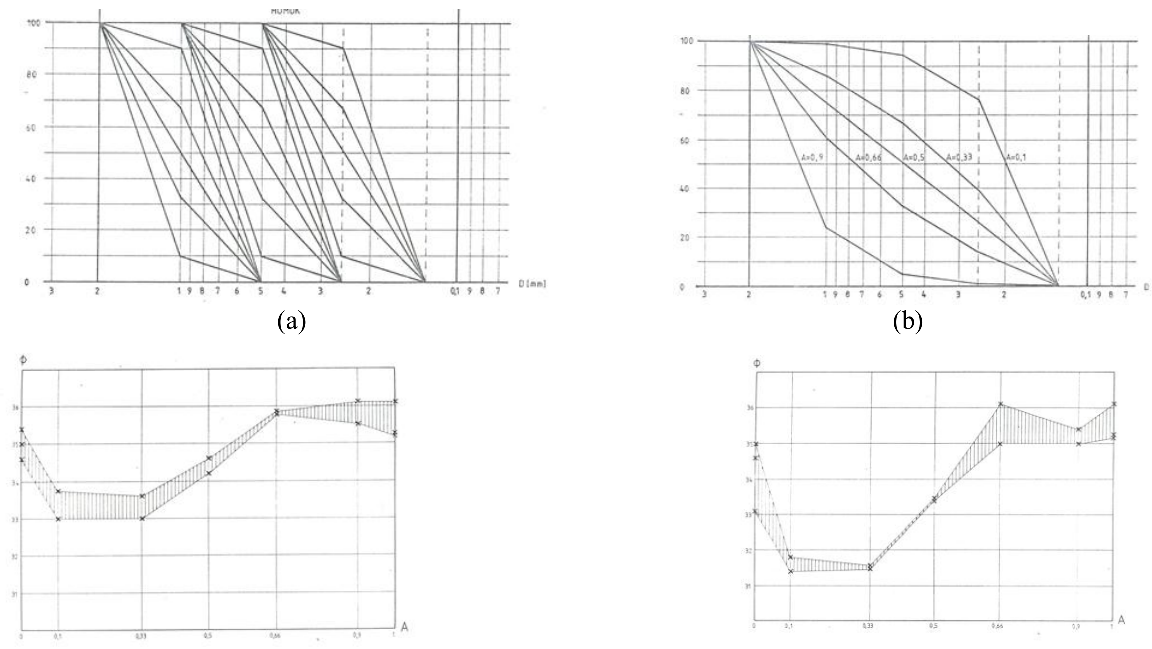
4.1. The structure of the grading curve space

Representing the space of the grading curves with *N* fractions by an *N* -1 dimensional, closed simplex, the *A* = constant condition means *N* -2 dimensional parallel hyper-planes in the Euclidean space generated by the simplex. The *B* = constant condition in addition is related to an *N* -3 dimensional, concentric topological circle around the optimal point on the *A* = constant hyper-plane section. The optimal point is defined by the maximum of *B* for the given *A* value. The optimal point is close to the mass center of the *A* = constant hyper-plane simplex section.

The optimal grading curve is a kind of mean grading curve considering all grading curves with the same *A*. It is a fractal grading is with minimum arc length. Being the dimension of the optimal line (one) less than the dimension of the space of the grading curves (*N*-1), it is worthy to set up any relationship between the mean gradings (instead of the whole space of the grading curves) and a given physical parameter.

The grading entropy parameters are various statistical means. The base entropy S_0 is a kind of dimensionless mean log diameter, which is similar to d_m. Its normalized value the relative base entropy parameter *A* is a normalized mean log diameter, varying between 0 and 1 with a shift symmetry in the log diameter axis, the extremes are related to the minimum and maximum log diameters. *A* indicates the relative distance of the mean and the minimum log *d* value. The base entropy S_0 is similar to d_m. Its normalized value, the relative base entropy *A* is a continuous internal stability measure, the *A*<2/3 condition indicates internally (more) unstable soils (as *A* decreases).

The entropy increment ΔS and its normalized version *B* are log weighted generalized geometrical means of the x_j (j = 1, 2, 3...*N*), having maximum values of lnN/ln2 and 1/ln2, respectively. For those grading curves, in which all *N* fractions are well represented, the entropy increment is typically close to the maximum value. They reflect the actual effective number of fractions within the mixture (like the coefficient of uniformity c_u), and also reflect the degree of degradation.



(a) (b)

(c) (d)

Figure 6. The relation of critical state friction angle and the entropy coordinate *A* in case of optimal sand soil mixtures ([15]). (a) and (b) grading curves of the 2-and 4-fraction sand mixtures. (c) and (d) critical state friction angle of the 2-and 4-fraction sand mixtures as *A* varies between 0 and 1.

4.2. The internal structure in terms of fines

The influence of fine particles on sand force structure, from micro-structure point of view, was first presented by Mitchell (1976) [13] and then simplifed by Thevanayagam (1998) [14] for other mechanical responses of transition soils. In two similar studies, the boundary between the "fines-in-sand" and "sand-in-fines" micro-structure was examined, and the threshold fines content was found at around *A*=2/3 ([8, 9]), supporting the internal stability criterion of [1].

4.3. Critical state friction angle

Artificial - mixtures of natural sand grains (see Figure 6) were used to determine the critical state friction angle in [15]. The four fractions were: 0.125-0.25 mm, 0.25-0.5 mm, 0.5-1 mm, 1-2 mm (uniform distribution was assumed within the limits). The critical state friction angle measured on optimal 2-and 4-fraction soils are shown in Figure 6. Saturated, drained triaxial tests were made with sample dimension of 100mm diameter, 100 mm height until critical state. The samples were saturated after compaction. According to Figure 6, it was found that the critical state friction angle was dependent on *A* for each soil series in the same way. It can be noted that this dependence was similar to the dependence of the coordination number (see Table 3) but was not in agreement with the DEM critical state friction angle (see Table 3). Further research is suggested on this question, on the variation of the critical state friction angle in terms of the grading curve and the coordination number.

5. Conclusion

Originally the influence of fine particles on sand force structure, from micro-structure point of view, was examined [13, 14]. Lőrincz ([1]) generalized this idea to any grading curve using grading entropy parameter *A*, and connected it to internal stability of soils. The relative base entropy parameter *A* measures the distance between the mean and the minimum log diameters, varying between 0 and 1, it has a potential to be a criterion number for internal stability, based on the simple physical fact that if the mean grain diameter is large enough, then enough large grains are present in a mixture and these will form a stable skeleton.

It follows from this sound physical basis that the two grading entropy coordinate pairs (normalized or non-normalized), proposed by Lőrincz ([1]) together with the integer variable N (the number of the fractions), S_{0min-} and d_{min-} (the entropy and diameter of the smallest fraction) may characterize the internal stability related phenomena like piping, liquefaction in terms of the grading curves more effectively than the usual diameter values.

To establish relations between the entropy parameters and soil physical parameters are promising, real and DEM experiments are suggested to be performed and reevaluated in further research. Also, to describe soil degradation, soil modification, compaction or breakage both in laboratory and in nature condition, the grading entropy theory is useful tool.

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