Dynamic Behaviour of a Geotechnical Seismic Isolation System with Rubber-Sand Mixtures to Enhance Seismic Protection

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Abstract: A Geotechnical Seismic Isolation (GSI) system is proposed in this study based on the use of Rubber-Soil mixtures (RSm) to facilitate the benefits of dynamic soil-foundation structure interaction. The latter is possible due to the lower stiffness and greater capacity to dissipate energy of RSm. However, the research done on RSm has been limited to the element scale context. In this study, the dynamic response of a modified soil foundation has been investigated by adding soft zones comprising RSm. A 1g shaking table was used to apply a sequence of sinusoidal excitations to a soil-lumped mass system. The results have shown that the rubber addition results in a reduction of the amplification at frequencies higher than the system natural frequency. This change in the dynamic response is due to the shift in the natural frequency and the dampening of the peak output accelerations. This study shows thus that an alternative design consideration with bagged soft zones, adjacent to the soil foundation, can offset the incoming disturbances and hence could protect both new and existing constructions.

Keywords: vibration isolation efficacy, shaking table test, geotechnical seismic isolation system, 1g model conditions.

1. Introduction

Earthquakes are one of the deadliest natural disasters. Since 1990, 17 major earthquakes (M > 7.0) have occurred every year and about 27,000 people have died in seismic events annually (Spence and Scawthorn, 2011). The majority of these casualties occur in highly populated urban areas in the developing world. This is mainly due to, amongst other reasons, residential buildings constructed with low quality materials and inadequate town planning (Guha-Sapir et al., 2016). Base isolation mechanisms have been successfully used in the past (Kelly and Van Engelen, 2015), however, they are difficult to deploy systems that can be unaffordable for low-to-medium rise buildings in the developing world.

The successful attenuation of surface waves, i.e. Rayleigh, together with the feasibility of their installation and maintenance has granted artificial barriers to be a commonly preferred vibration isolation system (Murillo, 2009; Woods, 1964). Existing research has proposed to transfer the mechanism behind the concept of wave screening and thus mitigate body waves transmitted through the soil for seismic protection (Lombardi, 2012; Kirtas et al., 2009). This can be achieved by introducing soft barriers, e.g. comprising geofams, reflecting part of the incident waves and subsequently attenuating the energy associated with body waves (Brennan et al., 2019; Flora et al., 2018; Nappa et al., 2016).

Alternatively, Geotechnical Seismic Isolation (GSI) systems seek to modify the soil by means of introducing flexible or sliding interfaces directly in contact with geological sediments, e.g. geosynthetic liners and wave barriers (Tsang, 2008). Within this field, Rubber-soil mixtures (RSm) have been recommended to mitigate the action of seismic motions due to its low stiffness and relatively high damping capacity (Tsang et al., 2020; Tsang and Pitilakis, 2019; Hazarika et al., 2008). The complexity of understanding the response of RSm under loading stems from adding "soft" particulate rubber. The material behaviour has been studied as a combination of stiff-soft particles which interact at a microscopic level and may influence the macroscopic behaviour depending on aspects such as rubber amount or size ratio between sand/rubber particles (Fonseca et al., 2019; Kim and Santamarina, 2008). The dynamic behaviour of RSm has been studied (Pistolas et al., 2018; Mashiri et al., 2016; Anastasiadis et al., 2012), however, most of these studies are limited to studying the mixture on the basis of soil element tests.

Few investigations have studied the performance of RSm under dynamic disturbances via 1g testing. This is analysed in the literature in terms of vibration isolation efficiency (Bandyopadhyay et al., 2015; Xiong and Li, 2013; Kaneko et al., 2013), i.e. ratio between (surface) output and input acceleration. The findings from these investigations showed a reduction of both horizontal and vertical accelerations at the surface level attributed to the increase in material damping. The isolation efficiency of RSm appeared to increase with more rubber as well as altering geometrical aspects, e.g. increasing the thickness or depth of RSm layer. Similar findings were shown in full-scale models with RSm using finite element analysis in which the RSm layer thickness was the main factor affecting the stress wave attenuation (Tsang and Pitilakis, 2019; Brunet et al., 2016; Tsang et al., 2012).

One common element to the previous studies is the material disposition. Hence, a unique design configuration has been typically adopted whereby continuous horizontal RSm layers are constructed underneath a scaled structure. This design incurs in two issues: a) a limiting vertical load on top of the foundation due to the potential static settlement of particulate rubber and b) it precludes its implementation to existing constructions.

As a means to overcome these issues, this study seeks to gain further understanding on the influence of soft zones, using bags comprising RSm, on the dynamic behaviour of a small-scale model subject to cyclic loading. A 1g shaking table was used to apply a sequence of sinusoidal excitations to a soil-structure system. The soil model was devised to allow zones of sand or RSm to be packed adjacent to a foundation. The stiffness of the modified RSm zone was controlled by the RSm composition. The aim of these experiments was to reveal the system natural frequency, amplification, damping capacity, as well as changes in resonance of the foundation system when using soft zones.

2. Materials and their dynamic properties

The RSm used in this study comprises a coarse rounded to sub-rounded Leighton Buzzard sand, with a sphericity and roundness of 0.65 and 0.78, respectively. The main material characteristics are average particle size d_{s50}=0.85 mm; coefficient of uniformity, C_u=1.25; and specific gravity, G_s=2.68. The rubber is obtained from car tyre sidewalls. Once all steel belts were removed, the rubber was devulcanised and then mechanically shredded. Compared to larger tyre shreds or chips, they are classified as rubber fibres in accordance with ASTM D6270 (2012). Rubber fibres used in this study present an average particle size d_{R50}=1.62 mm, and an aspect ratio between 2 and 6 (Figure 1a). Rubber G_s is taken to be 1.12 (Edil and Bosscher, 1994). The rubber thickness has been limited to 1 mm and exhibits a sphericity and roundness of 0.18 and 0.82, respectively. The particle size distribution for both sand and rubber are shown in Figure 1b. Specimens were prepared and tested adding sand and rubber amounts: χ = 0%, 10%, 20%, 30% and 40% by mass.

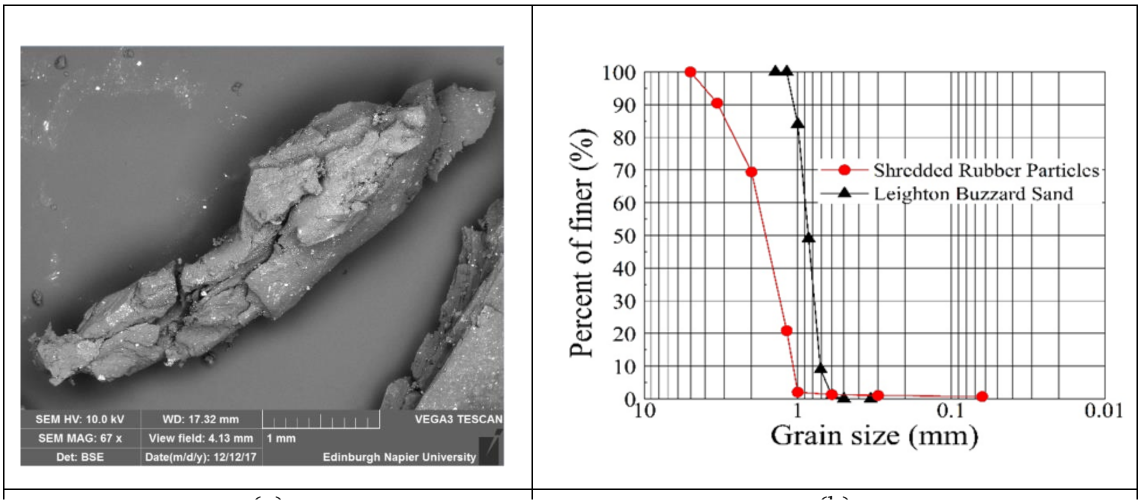


Figure 1 - a) Microscopic image of shredded rubber and b) Particle size distribution of sand and rubber

3. Experimental set-up

A timber box (0.285m x 0.185mx 0.185m) containing a lumped 3 kg mass located atop a central foundation silicon pillar (Figure 2) was used for this study. The sides of the timber box are rigidly fixed, i.e. bolted, to restrain any movement. The dynamic oscillation of the unsupported lumped mass and foundation column has been designed to mimic that of a Single Degree of Freedom (SDOF) system. The space between the foundation column and inside walls of the shake box in the direction of oscillation was packed with small bags containing particulate soils. This study seeks to compare the cyclic behaviour of a typical site adding to the bags: i) only sand, being a stiffer foundation, and ii) RSm with different rubber contents, i.e. soft barriers.

A very rigid silicon column was added underneath the lumped mass to ensure that the dynamic properties of the base did not change over time whilst providing a support to the top accelerometer. Thus, only the bags laterally added to the silicon column could lead to changes in the dynamic behaviour of the proposed GSI by increasing rubber content.

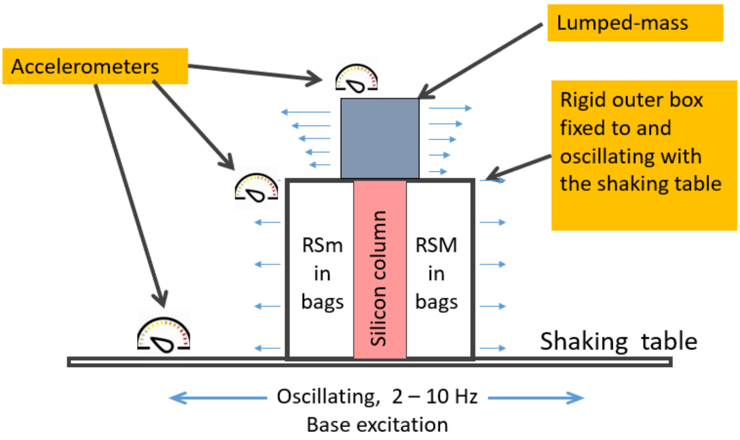


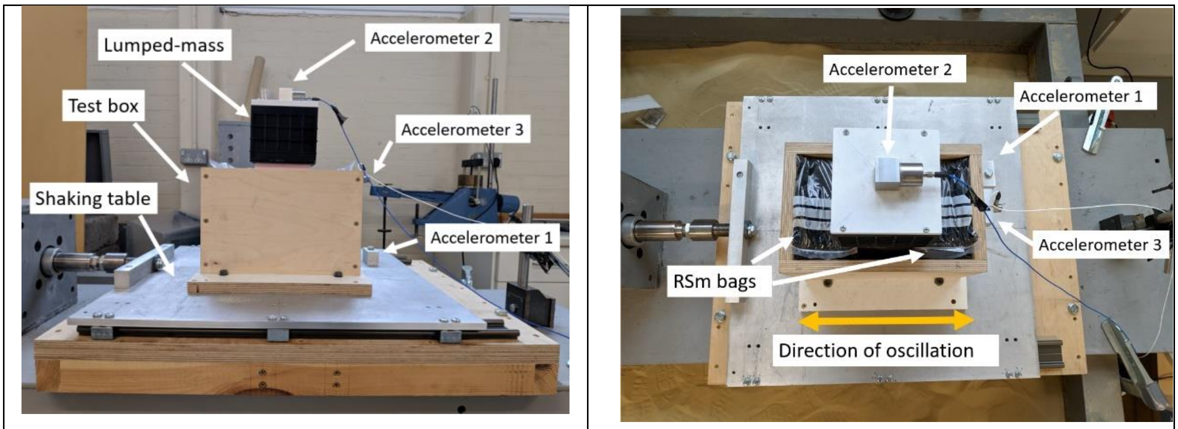
Figure 2 - Experimental set-up for shaking table test

The experimental testing was performed on a custom-built 0.5m by 0.5m shaking table (Figures 2 and 3). The table was shaken sinusoidally in one horizontal direction using an Electromechanical Cylinder (EMC) 80-32 manufactured by Rexroth. The horizontal response was measured with accelerometers mounted on the base of the table (A_1), and atop the lumped mass system (A_2). Additional measurements were taken on the outer side of the box (A_3) to conduct a system calibration test. Vertical accelerations were not measured on the principle of aiming to analyse a SDOF response.

The installation of the soft zones, adjacent to each side the silicon column (Figure 3), predetermined the specimen preparation. This is one of the main variations compared to the existing literature. Previous studies would place in various steps the horizontal RSm layers prior to the construction of the modelled structure, hence there was no risk of suffering from a significant segregation (Bandyopadhyay et al., 2015; Kaneko et al., 2013).

However, given the vertical disposition of the soft zones proposed in this study, if the mixture was to be added as a single soil, segregation would be expected as reported in previous studies (Pistolas et al., 2018). To prevent this, the solution consisted of bagging a constant RSm volume (140 cm3) into small plastic bags of 140mm by 90mm and adding 2.5 % moisture to the mixture. In contrast to existing studies, the latter design configuration would have the advantage of protecting existing constructions. Hence practical applications may extend to retrofitting foundations, well beyond the limited application to new buildings (only) as discussed by previous research.

The gravimetric proportion of rubber in the mixture, χ = 10%, 20%, 30% and 40%, was the main material descriptor. Bags were packed on the opposing faces of the foundation column that lie normal to the direction of horizontal oscillation. Fourteen bags were manually compacted, and they were installed acting as soft zones or walls up to a height of 185 mm, that is 55 mm below the top of the shake box rim. It is important to note that the presence of gaps in the RSm bags together with the gaps that inevitably form between bags means that volumetric state measures, such as void ratio, are of limited validity, and any attempts to define the volumetric state would be only indicative.



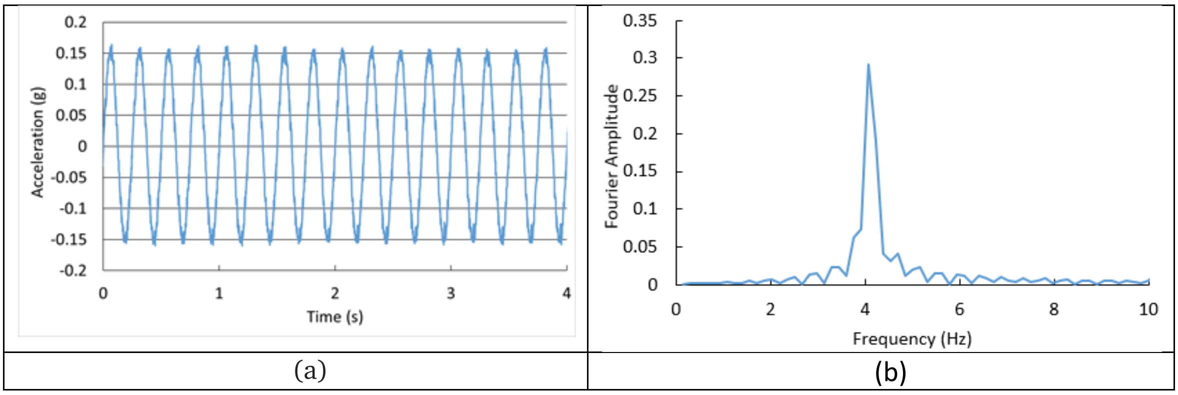
(a) (b)

Figure 3 - a) Front and b) plan view of rigid box

4. System calibration and test procedures

The objective of this manuscript is to explore the acceleration-time histories on top of a lumped-mass whilst modifying the soil foundation. Hence, the idea is to compare the design configurations herein proposed and thus elucidate the influence of adding higher rubber contents to the bagged soft zones on the cyclic performance of the system. For this reason, 1-g scaling factors have not been considered in this investigation and the values herein presented should not be directly related to any prototype scale test.

A system calibration test was performed to check the performance of the experimental set up before starting the experimental work. This is done to prove that any amplification generated by the system is negligible. The base motion used for the system calibration consists of 4-s horizontal sinusoidal motions with a constant displacement of 2.5mm at 4Hz frequency, which creates peak accelerations of 0.16g, as shown in Figure 4a. These sinusoidal motions correspond to the ones recorded by the accelerometer placed on the table (A_1). The response of the accelerometer outside the box (A_3) is shown in Figure 4b. It is observed that no significant amplification is recorded by the accelerometer and there is a predominant frequency of 4Hz, as it should be given the applied horizontal accelerations.



(a) (b)

Figure 4 - a) Base motion during system calibration (A_1), b) response of A_3 during system calibration

Regarding the test procedures, the shaking table tests ’sweep’ for the natural frequency of the “stiff” foundation, i.e. with sand only in the adjacent bags, and when installing the soft barriers, i.e. RSm. Sweeps were performed at integer frequency intervals from 2 to 10 Hz by subjecting the box to forced harmonic motions for 4 seconds. Oscillation displacement in each test was kept constant throughout the sweep, by control of velocity.

The tests were conducted into two stages:

- *Stage 1:* a sequence of 10 tests at increasing integer frequency (2-10 Hz). Notwithstanding earlier remarks about the gaps in RSm bags, Stage 1 was conducted to reduce the voids and thus densify the mixture prior to applying the fine sweep analysis. Stage 2 commenced then when there was no change in the system height.
- *Stage 2:* a sequence of tests at decreasing integer frequency (9-2 Hz). A limited range fine sweep at frequency intervals of 0.25 Hz were centred on the peak amplitude frequency. This served to improve the resolution around what had been revealed to be the resonant frequency.

Time domain analysis was applied to every sweep sequence as a means of evaluating the maximum response of the studied soil-structure system and compare it to the input motion over time. Using the peak horizontal accelerations, frequency domain analysis was after adopted to determine the amplification ratio (A_r) obtained from the ratio output response/input motion with respect to the studied frequency range (Chopra, 2011) as follows:

$$A_r(f = x) = \frac{A_{building}}{A_{table}} \tag{1}$$

Where, f = input frequency and x = 2 - 10Hz , $A_{building}$ is the peak acceleration recorded on top of the lumped-mass (A_2), and A_{table} is the peak acceleration recorded on the table (A_1).

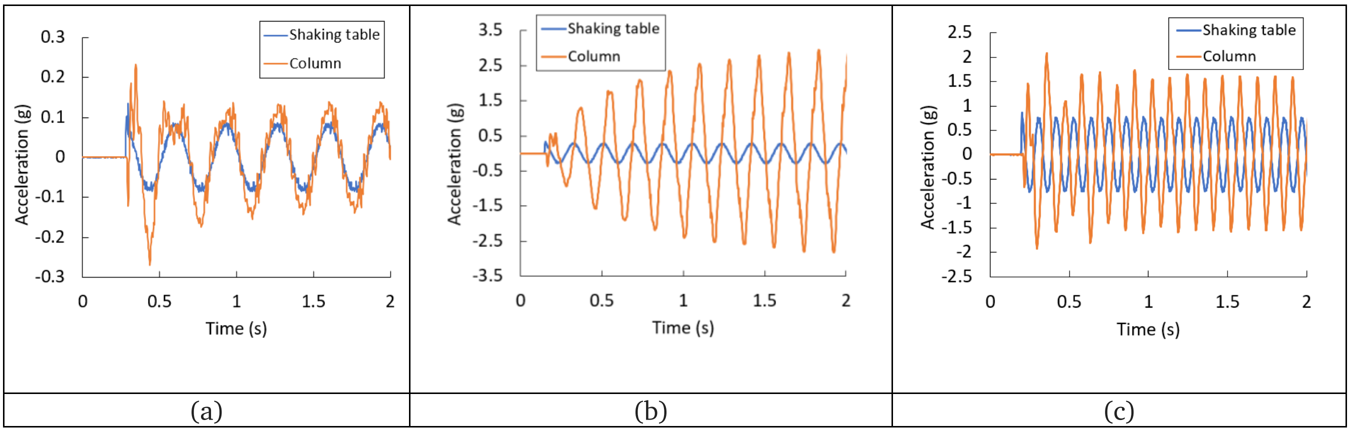
This analysis was undertaken to understand whether the soil-structure system amplified, i.e. $A_r > 1$, or maintained, i.e. $A_r = 1$, the incoming disturbances. A direct comparison has thus been created in this study between i) a stiff foundation, with sand only ($\chi = 0\%$), and ii) the modified site with soft inclusions by adding RSm ($\chi = 10 - 40\%$).

5. Results

5.1 Acceleration-time histories - Sand only

Figure 5 presents a typical set of acceleration-time histories for the shaking table (input) and on the lumped-mass, atop the silicon column (output), when adding only sand to the bags, which describe a different response in relation to the control frequency: 3 Hz, 5.5 Hz and 9 Hz.

At low frequencies (Figure 5a), there is little amplification, and no phase lag is observed (± 0.1 g). This behaviour changes at medium frequencies (Figure 5b), in which an amplification of the input signal was recorded with the number of cycles and resonance was reached at f = 5.5Hz (± 2.85 g). At higher frequencies (Figure 5c), it is observed that the lumped-mass experiences an amplification of the input excitation. However, the evolution of the output value remains constant with the number of cycles. The increase in the output acceleration is due to the rise in the input frequency and also input acceleration. These results demonstrate that the dynamic response experienced by the central column mimics that of a SDOF system and the peak output accelerations change with the input frequency.



(a) (b) (c)

Figure 5 - Sand only. Transmitted accelerations on table and column at a) f = 3 Hz b) f = 5.5 Hz and c) f = 9 Hz

5.2 Amplification ratio - RSm installations

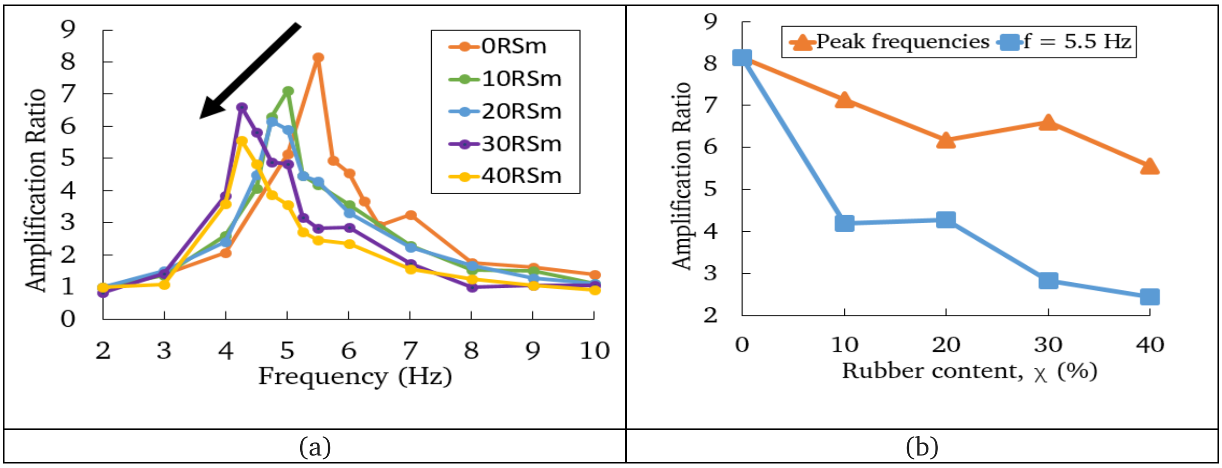
The dynamic response of the modified foundation has been studied by comparing the amplification ratio in the frequency range studied (2-10 Hz). For this, the output response has been derived from the peak accelerations for sand only (Figure 5) and all RSm specimens ($\chi = 10$–40%). Figure 6a shows that the output acceleration for all RSm remains relatively close to the input motion, i.e. $A_r = 1 - 1.5$, at low (2 Hz) or high (10 Hz) frequencies, and it increases as the input frequency approaches the system natural frequency, as found with the stiffer foundation in Figure 5. Thus, the acceleration recorded on top of the column is observed to increase by 6 to 9 times ($A_r = 6$-9) the input around the system natural frequency. This is a significant amplification of the vibrations if compared to the literature (Bandyopadhyay et al., 2015; Xiong and Li, 2013; Kaneko et al., 2013). The reason for this significant amplification is due to the configuration adopted in this study with the use of a lumped mass system.

Figure 6a reveals how the presence of RSm inclusions affect the amplification conditions on top of the column. Considering the peak frequency corresponds to the frequency at which resonance is reached, Fig. 6a shows that the natural frequency of the foundation-modified soil system decreases with rubber. This is in line with previous studies (Nappa et al., 2016; Kaneko et al.,

2017) that demonstrated a lengthening of the system period when adding horizontal soft layers, causing a shift in the response of the modified soil.

Based on the results corresponding to the dynamic behaviour found in Bernal-Sanchez et al. (2019, 2018), the addition of rubber results in a decay of the mixture shear modulus and, in turn, a lower resistance to deformation under cyclic loading. As the system maintains the same height and density, this leads to a reduction in the shear wave velocity and the natural frequency of the system. This explains why sand only reaches resonance at higher frequencies whilst the addition of rubber to the zones adjacent to the foundation results in a shift in the system natural frequency.

Figure 6b presents the change in the amplification ratio comparing the reference case (χ = 0%) and the RSm inclusions considering: i) f = 5.5 Hz, and ii) peak frequencies. In both cases a reduction in the amplification ratio is observed. The attenuation of the transmitted disturbances is more evident when comparing the amplification ratio at the natural frequency of the stiff foundation (f = 5.5 Hz) passing from $A_r = 8$ with sand only to $A_r = 2.5$ with 40% RSm. This is consistent with the reduction in the system natural frequency. The reduction in the amplification ratio for the peak frequencies would be, on the other hand, attributable to an attenuation (i.e. dampening) of the input motion. This finding reveals that the alternative design consideration proposed in this study, i.e. lateral RSm installations, can also offset the incoming disturbances whilst also attenuating the system peak accelerations.



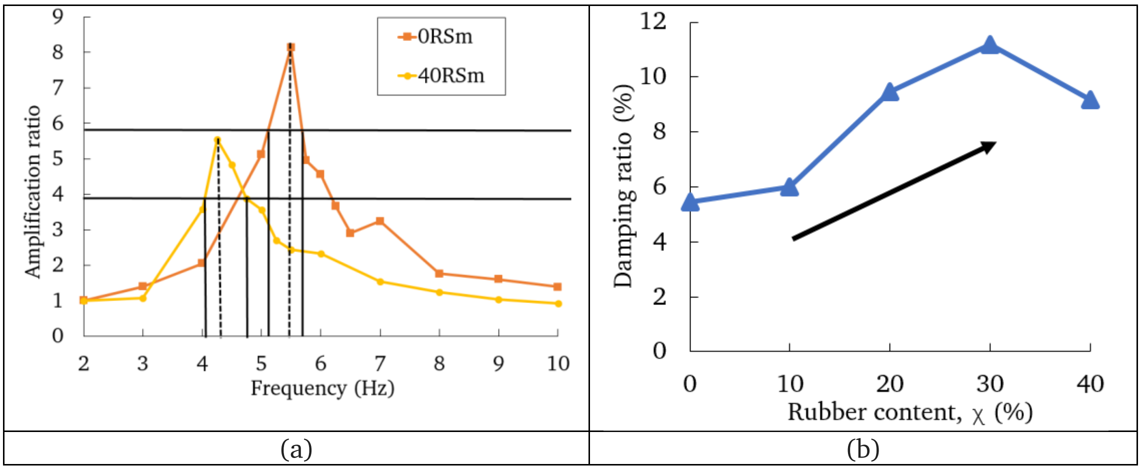
(a) (b)

Figure 6 - Amplification ratio vs (a) frequency, and (b) rubber content

5.3 Damping capacity

Time domain analysis, using free vibration methods, rely on the analysis of a unique signal whilst decreasing strength and it is postulated by Signes et al. (2017) difficult to isolate from other dynamic effects, i.e. boundary reflections. This is not seen as an optimal approach for unbound granular materials, i.e. RSm, which could introduce additional wave reflections and subsequent high frequency noise. Instead, the half-power bandwith method (Papagiannopoulos and Hatzigeorgiou, 2011) is commonly adopted to calculate the damping ratio (ξ) of SDOF systems, which is the case of the lumped-mass soil foundation herein presented.

For comparison purposes, Figure 7a shows the bandwidths corresponding to the reference case and 40% RSm, where there is an evident reduction in the system natural frequency. A narrower band is visually observed with sand only, revealing a lower damping capacity. The damping ratio of the modified foundation has been determined for all rubber contents based on the half-power bandwith (Figure 7b). The results point to an improvement in damping capacity with rubber content, doubling the damping ratio at χ = 30%. Adding rubber then leads to an increase in the energy dissipation capacity of the foundation, reducing the output acceleration on the top of the lumped-mass. This would explain the decay observed in the amplification ratio at peak frequencies when adding rubber (Figure 6b).



(a)

(b)

Figure 7 - a) Half-power bandwith for sand and 40% RSm, b) system damping capacity

5.4 Vibration isolation efficacy

The change in amplification ratio has been determined between each RSm scenario and the reference case ($\chi = 0\%$), i.e. $\frac{A_r \text{ RSm}}{A_r \text{ sand}}$, for the studied frequency range. In Fig. 8, two frequency bands are shown: a) before, and b) after the resonant frequency of the reference case. For the former, the amplification ratio when adding RSm installations is greater than with a stiff foundation hence there is a greater amplification of the input signal. The amplification appears to be more pronounced at $\chi = 30$ - 40% due to the greater shift in the natural frequency of the system. Hence, adding soft installations can worsen the transmission of initial disturbances, resulting in greater peak accelerations. A practical implication of this could be the occurrence of exacerbated amplifications at low-frequency earthquakes or around slender buildings.

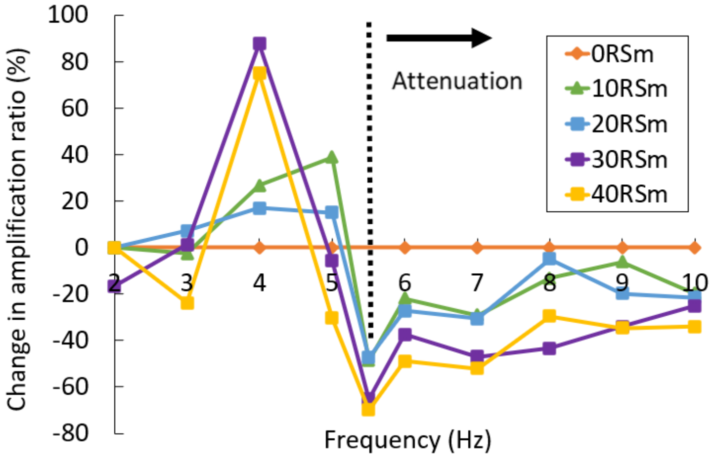


Figure 8 - Change in amplification ratio with RSm compared to reference case

At frequencies higher than 5.5 Hz, the addition of rubber points to a decay in amplification ratio compared to the reference case, which coincides with the attenuation in horizontal accelerations recorded on top of the column. This phenomenon occurs at any rubber content, but it is more accentuated at $\chi = 30$ - 40% where reductions of up to 70% in amplification ratio are observed. Therefore, the inclusion of soft zones successfully offset the expected peak output accelerations. This means in practice that modifying the soil foundation by adding RSm could be adopted in areas where predominantly high-frequency earthquakes are expected or to protect low-rise buildings.

6. Conclusions

In this study, the influence of bagged RSm installations on the dynamic behaviour of a lumped-mass foundation system has been investigated using a 1g shaking table. The major findings obtained in this study are as follows:

- The addition of RSm inclusions to the foundation alters the amplification conditions of the lumped mass (atop the column) by shifting the system natural frequency.
- A reduction in the amplification of the system is reached with the addition of greater rubber contents and this is attributable to both i) the attenuation (dampening) of the peak values and ii) the shift in the natural frequency with the reduction in stiffness.
- The vibration isolation efficacy, .e. $\frac{A_r\ \mathrm{RSm}}{A_r\ \mathrm{sand}}$, has shown a lower amplification of the input signal at frequencies higher than the fundamental frequency of the stiffer (only sand) foundation, whereas the opposite was observed at lower frequencies. This suggests that adding RSm could attenuate the transmission of seismic disturbances around low-rise buildings, whilst it could worsen the amplification of slender buildings.
- The attenuation in the peak horizontal accelerations with RSm is linked to the increase in the system damping capacity, which is shown to improve with rubber content.
- The results of this study provide an alternative design configuration to the previously adopted (in the literature) horizontal RSm layers underneath the soil foundation. Therefore, the solution herein proposed could be considered for protecting not only new constructions but also retrofitting existing foundations against seismic disturbances.

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