INFLUENCE OF GRADING ON SHEAR STIFFNESS – THE SIGNIFICANCE OF ACCURATE DESCRIPTION OF PARTICLE SIZE DISTRIBUTIONS

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1. Introduction

Various relationships between grading parameters and shear stiffness have been proposed. For example, Wichtmann & Triantafyllidis [1] to relate the coefficient of uniformity ($c_u=d_{60}/d_{10}$), to the shear stiffness. Menq [2] also used the mean particle diameter (d_{50}) in addition to c_u to estimate the shear stiffness. Considering that parameters such as c_u and d_{50} only refer to specific fractions of the PSD, Sun et al [3] proposed a new (c_g) parameter that considers the entire shape of the PSD. They then used c_g to estimate shear stiffness values and argued that this parameter was better than other existing ones to predict shear stiffness values.

Although, the work by Sun et al [3] considers the entire PSD for the calculation of c_g, it shares a deficiency with the existing ones; they cannot easily consider variations in PSD with time. This is important because in geotechnical applications there are phenomena such as internal erosion, dissolution, degradation and crushing that produce changes in PSD and therefore changes in shear stiffness. Furthermore, c_u, d_{50} and c_g relate only to the shape of the PSD, but have no link with physical properties and therefore lack a definite physical meaning that can be used to interpret stiffness evolution.

2. Grading entropy coordinates (and inter-particle contact force entropy coordinates)

In the context of grading, the meaning of the term ‘entropy’ refers to a ‘multiplicity of the microstates of a system’ [5]. Statistical entropy allows the microscopic configuration of a system to be described, in this case the representation of a particle size distribution, as a coordinate pair, which then plots as a single point in a grading entropy diagram. A vectorial depiction of a change in grading, rather than a family of distribution curves, is then available. The grading entropy coordinates are the relative base entropy A, and the normalised entropy increment B. The parameter A is a measure of the skewness or symmetry of a particle size frequency distribution, while B is a measure of the kurtosis or peakiness of a particle size frequency distribution. Further details are discussed elsewhere [6-7].

As a mathematical concept, the coordinates A and B can also be used to evaluate the characteristics of the magnitude of normal contact forces which are obtained from DEM simulations. It can be postulated that these force entropy coordinates may be related to grading entropy, but this is out of scope for this study. The emphasis here is not on the processes that the methodology can represent (i.e. dissolution, erosion, breakage, etc.) but in the concept of entropy coordinates and their relationship with initial shear stiffness.

3. Initial shear stiffness and its relationship with grading entropy coordinates.

Following the ideas by [3] Figure 1 illustrates the evolution of the normalised initial shear stiffness obtained from resonant column tests on silica sands by [1]. The different symbols represent different mean particle diameters (d_{50}). There are two sets of data, one for tests at 100 kPa confining pressure (above) and the other one at 400 kPa (below). Note that a void ratio function $[=(2.17-e)^2/(1+e)]$ has been used. Independently of stress level, there is a linear relationship between the relative base entropy (A) and initial shear stiffness. This is in contrast to the work in [1] and [3], where non-linear relationships have been found using a similar approach. Physically however, Figure 1 illustrates that G is a function of the symmetry of the PSD.

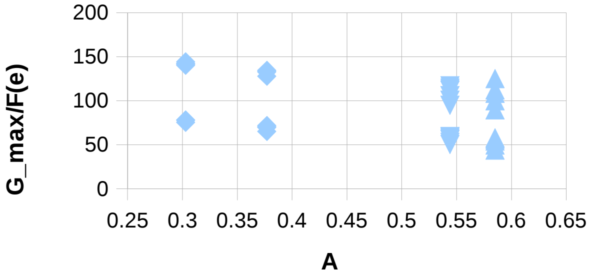

Figure 1. Relationship between relative base entropy and initial shear stiffness

4. Contact force entropy and stress-strain response.

The results of a set of DEM simulation of triaxial compression tests with different initial densities is presented in Figure 2. There is a significant amount of information of such a diagram, here it is highlighted that each test starts at different initial points (dependent on density); and that all tests converge into a single point at large strain. In other words, the magnitude of contact forces and entropy coordinates are suitable to understand the mechanics of critical states.

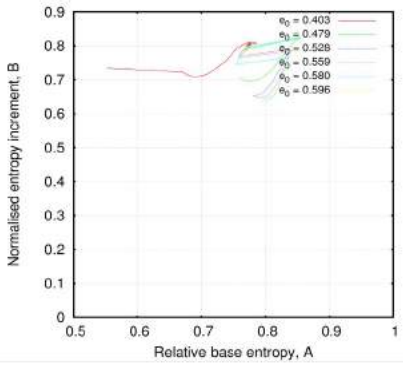

Figure 2. Entropy diagram for a set of DEM simulations of triaxial compression tests

5. References

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